

The Use of GARCH Models in VaR Estimation

Timotheos Angelidis, Alexandros Benos^{*}
and
Stavros Degiannakis[†]

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Abstract

We evaluate the performance of an extensive family of ARCH models in modelling daily Value-at-Risk (VaR) of perfectly diversified portfolios in five stock indices, using a number of distributional assumptions and sample sizes. We find, first, that leptokurtic distributions are able to produce better one-step-ahead VaR forecasts; the choice of sample size is important for the accuracy of the forecast, whereas the specification of the conditional mean is indifferent. Finally, despite the claims for the contrary, a different and specific structure of ARCH model produces the most accurate VaR forecast for each index portfolio.

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^{*}Dept. of Banking and Financial Management, University of Piraeus, 80, Karaoli & Dimitriou street, University of Piraeus, Piraeus GR-185 34, Greece. Email: abenos@unipi.gr.

[†]Dept. of Statistics, Athens University of Economics and Business. The usual disclaimer applies.

1 Introduction

Following the increase in financial uncertainty in the 90's, resulting to famous financial disasters, (see Jorion (1997), Crouhy et al. (2001), Dowd (1998) for more information), there has been intensive research from financial institutions, regulators and academics to better develop sophisticated models for market risk estimation. The most well known risk measure is Value-at-Risk (VaR) which refers to the amount of money a portfolio is likely to lose over some predefined period and at a given confidence level.

In order to estimate VaR, we can use either non-parametric or parametric methods. The historical simulation methodology belongs to the former family and, instead of making distributional assumptions on portfolio returns, it uses realized past returns to estimate the given percentile. Its main advantage is the parsimony in assumptions needed: one only needs to suppose the distribution of portfolio returns is constant over the estimation period. On the other hand, it does not allow for changing variance-covariance matrix of the returns and, by definition, its estimations are sample dependent.

The most common parametric method is the analytic one, also known as the Variance-Covariance method, which makes specific distributional assumptions on returns and, therefore, easily computes (analytically or numerically) the corresponding percentile. Although, for its simplicity, the normal distribution is widespread, it cannot effectively describe the tails of portfolio returns, due to leptokurtosis. Hence, other distributions, such as the Student-t and the Generalized Error Distribution (GED), are applied instead, modelling tail thickness by a parameter, called "degrees of freedom".

The above models do not, however, incorporate the observed volatility clustering of returns, first noted by Mandelbrot (1963). The most popular model taking account of this phenomenon is the Autoregressive Conditional Heteroscedasticity (ARCH) process, introduced by Engle (1982) and extended by Bollerslev (1986).

The purpose of our paper is to implement several volatility models under three different distributional assumptions of returns and four historical sample sizes in order to estimate the 95% and 99% one-day VaR for five completely diversified index portfolios (S& P500, Nikkei 225, FTSE 100, CAC 40 and DAX 30). The different distributions will allow the selection of a more realistic model for the return tails, while four sample sizes (500, 1000, 1500 and 2000 observations) will reveal the importance of past data. Although the use of all available data in forecasting volatility is common practice, we find out that, in some cases, a restricted sample size could generate more accurate one-step-ahead VaR forecasts as it could incorporate changes in trading behaviour more easily. For robustness purposes, we have used five different stock indices, to avoid results dependent on a specific financial market. Combined with three different interpretations of the ARCH family (GARCH, TARARCH and EGARCH) and more than 1800 one-step-ahead VaR estimate, it brings the total of estimated models to more than 4 million ! Finally, two back testing criteria are implemented in order to investigate the performance of each technique.

Our study shows that the extended versions of GARCH fare better than the simple version in all indices, all distributional assumptions and both confidence levels. As expected, the leptokurtic distributions provide better estimators of VaR since they perform better in the low probability regions Value at Risk tries to measure.

The rest of the paper is organized as follows. Section 2 provides a description of the ARCH models, while the third section describes the evaluation framework for VaR estimates. Section 4

presents preliminary statistics for the dataset, explains the estimation procedure and presents the results of the empirical investigation of the estimated models for the five equity indices. Section 5 concludes.

2 Volatility Models

Let $y_t = \ln(S_t/S_{t-1})$ denote the continuously compound rate of return from time $t - 1$ to t , where S_t is the asset price at time t . We assume that the time series of interest, y_t , is decomposed into two parts, the predictable and unpredictable component $y_t = \mathbb{E}(y_t|I_{t-1}) + \varepsilon_t$, where I_{t-1} is the information set at time $t - 1$, \mathbb{E} is the conditional mean operator and ε_t is the unpredictable part, or innovation process. The conditional mean return is considered as a k -th order autoregressive process, $AR(k)$:

$$\mathbb{E}(y_t|I_{t-1}) \equiv c_0 + \sum_{i=1}^k c_i y_{t-i}.$$

The autoregressive process allows for the autocorrelation induced by discontinuous (or non-synchronous) trading in the stocks making up an index (Scholes and Williams (1977), Lo and MacKinlay (1988)). The unpredictable component, ε_t , can be expressed as an ARCH process in the following form:

$$\varepsilon_t = z_t \sigma_t,$$

where z_t is a sequence of independently and identically distributed random variables with mean zero and unit variance. The conditional variance of ε_t is σ_t , a time-varying, positive and measurable function of the information set at time $t - 1$. Note that, even though the innovation process for the conditional mean is serially uncorrelated, it is not time independent.

Engle (1982) introduced the ARCH(q) model and expressed the conditional variance as a linear function of the past q squared innovations

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2.$$

For the conditional variance to be positive, the parameters must satisfy $a_0 > 0$ and $a_i \geq 0$ for $i = 1, \dots, q$. Empirical evidence has shown that a high q must be selected in order to estimate the conditional variance properly. To circumvent this problem, Bollerslev (1986) proposed the generalized ARCH, or GARCH(p, q), model:

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \sum_{j=1}^p b_j \sigma_{t-j}^2, \quad (1)$$

where $a_0 > 0$, $a_i \geq 0$ for $i = 1, \dots, q$, and $b_j \geq 0$ for $j = 1, \dots, p$. If $\sum^q a_i + \sum^p b_j < 1$, then the process ε_t is covariance stationary and its unconditional variance is equal to

$$\sigma^2 = \frac{a_0}{1 - \sum^q a_i - \sum^p b_j}.$$

A special case of the GARCH family is the Exponentially Weighted Moving Average (EWMA) alternative, used by the company RiskMetricsTM, when they introduced the analytic VaR methodology. The volatility forecast is the weighted average of the previous period's forecast and the

current squared return. They model return variance as an exponentially declining process $\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1 - \lambda)\varepsilon_{t-1}^2$, a special case of GARCH(1,1) with a zero intercept and the two remaining parameters summing to one. RiskMetricsTM use $\lambda = 0.94$ for daily data and go 75 data points backwards in their estimation horizon.

The GARCH(p,q) model successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering, as noted by Mandelbrot (1963): "... large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes...". On the other hand, the GARCH structure present some drawbacks on implementation, since it (a) requires large numbers of observations to produce reliable estimates, and (b) may be unstable out of sample. It also imposes important limitations on the theoretical approach since variance depends on the magnitude and not on the sign of ε_t , which is somewhat at odds with the empirical behaviour of stock market prices where a leverage effect may be present.¹ The term leverage effect, first noted by Black (1976), refers to the tendency for changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to bad news, ($\varepsilon_t < 0$), and to fall in response to good news ($\varepsilon_t > 0$).

In order to capture the asymmetry exhibited by the data, a new class of models was introduced, termed the asymmetric ARCH models. The most popular model proposed to capture the asymmetric effects is Nelson's (1991) exponential GARCH, or EGARCH(p,q), model:

$$\ln(\sigma_t^2) = a_0 + \sum_{i=1}^q \left(a_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^p (b_j \ln(\sigma_{t-j}^2)). \quad (2)$$

In contrast to the GARCH model, no restrictions need to be imposed on the model estimation, since the logarithmic transformation ensures that the forecasts of the variance are non-negative. The parameters γ_i allow for the asymmetric effect. If $\gamma_1 = 0$ then a positive surprise, $\varepsilon_t > 0$, has the same effect on volatility as a negative surprise, $\varepsilon_t < 0$. The presence of leverage effect can be investigated by testing the hypothesis that $\gamma_1 < 0$.

The number of possible conditional volatility formulations is vast. The threshold GARCH, or TAR(p,q), model is one of the widely used models:

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \gamma_1 \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^p b_j \sigma_{t-j}^2, \quad (3)$$

where $d_t = 1$ if $\varepsilon_t < 0$, and $d_t = 0$ otherwise. The TAR(p,q) model allows a response of volatility to news with different coefficients for good and bad news.

A wide range of ARCH models proposed in the literature has been reviewed by Bollerslev et al. (1992, 1994), Bera and Higgins (1993), Hamilton (1994), and Degiannakis and Xekalaki (2003). Engle (1982) who introduced the ARCH process assumed a normal density. Bollerslev (1987), however, proposed the standardized t-distribution with $v > 2$ degrees of freedom

$$D(z_t; v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left(1 + \frac{z_t^2}{v-2}\right)^{-\frac{v+1}{2}}, \quad (4)$$

¹In such a case, a stochastic volatility model may be more appropriate. There is some evidence that these models may perform better than multivariate GARCH models (Danielsson (1996)) but they are also more difficult to estimate.

where $\Gamma(v) = \int_0^\infty e^{-x} x^{v-1} dx$ is the gamma function. The degree of freedom represents the parameter to be estimated. The t-distribution is symmetric around zero and, for $v > 4$, the conditional kurtosis equals $3(v-2)(v-4)^{-1}$, which exceeds the normal value of three, but for $v \rightarrow \infty$, the density function of the standardized t-distribution converges to the density function of the standard normal distribution. Nelson (1991), on the other hand, suggested the use of the generalized error distribution, or GED:

$$D(z_t; v) = \frac{v \exp(-0.5 |z_t/\lambda|^v)}{2^{(1+1/v)} \Gamma(v^{-1}) \lambda}, \quad v > 0, \quad (5)$$

where v is the tail-thickness parameter and $\lambda \equiv \sqrt{2^{-2/v} \Gamma(v^{-1}) \Gamma(3v^{-1})}$. When $v = 2$, z_t is standard normally distributed. For $v < 2$, the distribution of z_t has thicker tails than the normal distribution (e.g., for $v = 1$, z_t has a double exponential distribution) while for $v > 2$, the distribution of z_t has thinner tails than the normal distribution (e.g., for $v = \infty$, z_t has a uniform distribution on the interval $(-\sqrt{3}, \sqrt{3})$). Guermat and Harris (2002) applied the Student t-distribution and Longestay (1996) used a mixture of normal distributions. The Gram-Charlier type distribution (Lee and Tse (1991)), the generalized t-distribution (Bollerslev et al. (1994)), the skewed t-distribution (Lambert and Laurent (2000, 2001)), the normal Poisson mixture distribution (Jorion (1988)) and the normal lognormal mixture (Hsieh (1989)) among others are distributions that have been employed.

Maximum likelihood estimation is commonly used to estimate ARCH models. Under the assumption of independently and identically distributed standardized innovations, z_t , and for $D(z_t; v)$ denoting their density function, the log-likelihood function of $\{y_t(\theta)\}$ for a sample of T observations is given by:

$$\mathbb{L}_T(\{y_t\}; \theta) = \sum_{t=1}^T \left[\ln [D(z_t(\theta); v)] - \frac{1}{2} \ln(\sigma_t^2(\theta)) \right], \quad (6)$$

where θ is the vector of the parameters that have to be estimated for the conditional mean, variance and density function. The maximum likelihood estimator $\hat{\theta}$ for the true parameter vector is found by maximizing (6). Hence, we have the following result for the log-likelihood function applied to a sample of T observations:

1. For normally distributed standardized innovations:

$$\mathbb{L}_T(\{y_t\}; \theta) = -\frac{1}{2} \left[T \ln(2\pi) + \sum_{t=1}^T z_t^2 + \sum_{t=1}^T \ln(\sigma_t^2) \right]. \quad (7)$$

2. For standardized t-distributed innovations:

$$\begin{aligned} \mathbb{L}_T(\{y_t\}; \theta) = & T \left[\ln \Gamma \left(\frac{v+1}{2} \right) - \ln \Gamma \left(\frac{v}{2} \right) - \frac{1}{2} \ln[\pi(v-2)] \right] - \\ & \frac{1}{2} \sum_{t=1}^T \left[\ln(\sigma_t^2) + (1+v) \ln \left(1 + \frac{z_t^2}{v-2} \right) \right]. \end{aligned} \quad (8)$$

3. For GED distributed innovations:

$$\begin{aligned} \mathbb{L}_T(\{y_t\}; \theta) &= \sum_{t=1}^T \left[\ln\left(\frac{v}{\lambda}\right) - \frac{1}{2} \left| \frac{z_t}{\lambda} \right|^v - \right. \\ &\quad \left. (1 + v^{-1}) \ln(2) - \ln \Gamma\left(\frac{1}{v}\right) - \frac{1}{2} \ln(\sigma_t^2) \right], \end{aligned} \quad (9)$$

with all parameters as defined before. Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm (Marquardt (1963)). We use the quasi-maximum likelihood estimator (QMLE) since, according to Bollerslev and Wooldridge (1992), it is generally consistent, has a normal limiting distribution and provides asymptotic standard errors that are valid under non-normality.

We now present the forecast recursion relationships to compute the one-step-ahead conditional variance forecast, $\hat{\sigma}_{t+1|t}^2$. Specifically, for the GARCH(p,q) model, the relationship is:

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q a_i^{(t)} \varepsilon_{t-i+1}^2 + \sum_{j=1}^p b_j^{(t)} \sigma_{t-j+1}^2. \quad (10)$$

For the EGARCH(p,q) model, we get instead:

$$\hat{\sigma}_{t+1|t}^2 = \exp \left(a_0^{(t)} + \sum_{i=1}^q \left[a_i^{(t)} \left| \frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right| + \gamma_i^{(t)} \left(\frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right) \right] + \sum_{j=1}^p b_j^{(t)} \ln(\sigma_{t-i+1}^2) \right). \quad (11)$$

Notice that the value of $\mathbb{E}|z_t|$ depends on the density function of z_t . For example, for the standard normal distribution, $\mathbb{E}|z_t| = \sqrt{2/\pi}$; for the Student t-distribution,

$$\mathbb{E}|z_t| = \frac{2\Gamma[(1+v)/2]\sqrt{v-2}}{1+(v-1)\Gamma(v/2)\sqrt{\pi}},$$

and for the GED distribution,

$$\mathbb{E}|z_t| = \lambda 2^{1/v} \frac{\Gamma(2/v)}{\Gamma(1/v)}.$$

However, the quantity $\varepsilon_{t-i+1} \sigma_{t-i+1}^{-1}$, both with and without the absolute value operator, for $i \geq 1$, can be straightforwardly computed by the model, as the values of both the innovation and its conditional standard deviation are available. Finally, the corresponding one-step-ahead conditional variance forecast in the case of the TARCH(p,q) model is:

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q \left[a_i^{(t)} \varepsilon_{t-i+1}^2 \right] + \gamma^{(t)} \varepsilon_t^2 d_t + \sum_{j=1}^p \left[b_j^{(t)} \sigma_{t-j+1}^2 \right]. \quad (12)$$

Hence, it is straightforward to compute the one-step-ahead VaR forecast, since under all distributions, we can compute the corresponding quantiles, which we then multiply by our conditional standard deviation forecast.

3 Evaluating the different approaches

Our objective is to test these different volatility forecasting techniques in a risk management environment. The quality of our VaR forecast and, naturally, anything else depending on these forecasts, depend on the quality of our volatility forecasts. We must, nevertheless, first choose a metric by which we will judge the “quality” of VaR forecasts. It is well known that there are many sources of error in VaR figures: sampling errors, data problems, inappropriate models or specifications, etc. All these factors will make our VaR estimate to often be biased. We must also tackle the fact that we never actually observe any realized VaR value after the event, since our model forecasts an inherently unobservable variable. We, therefore, have to monitor VaR forecasts and, afterwards, judge volatility models, not by checking whether our forecasts are realized, but whether they are consistent with subsequently realized returns given the confidence interval on which the forecasts were constructed in the first place.

3.1 Unconditional Coverage

Let $N = \sum_{t=0}^T I_{t+1}$ be the number of days over a T period that the portfolio loss was larger than the VaR estimate, where

$$I_{t+1} = \begin{cases} 1, & \text{if } y_{t+1} < \text{VaR}_{t+1}; \\ 0, & \text{if } y_{t+1} \geq \text{VaR}_{t+1}. \end{cases}$$

Hence, N is the observed number of exceptions in the sample. As argued in Kupiec (1995), the failure number follows a binomial distribution, $N \sim B(T, p)$, and consequently the appropriate likelihood ratio statistic, under the null hypothesis that the expected exception frequency $N/T = p$, is:

$$2 \ln \left[\left(1 - \frac{N}{T}\right)^{T-N} \left(\frac{N}{T}\right)^N \right] - 2 \ln \left[(1-p)^{T-N} p^N \right],$$

Asymptotically, this test is distributed as a χ^2 with one degree of freedom, while, in Table 1, we present the no rejection regions of N for various sample sizes and confidence levels. This test can reject a model for both high and low failures but, as stated in Kupiec (1995), its power is generally poor. So we turn to a more elaborate criterion.

Confidence level	Evaluation sample size			
	250	500	750	1000
5%	$7 \leq N \leq 19$	$17 \leq N \leq 35$	$27 \leq N \leq 49$	$38 \leq N \leq 64$
1%	$1 \leq N \leq 6$	$2 \leq N \leq 9$	$3 \leq N \leq 13$	$5 \leq N \leq 16$
0.5%	$0 \leq N \leq 4$	$1 \leq N \leq 6$	$1 \leq N \leq 8$	$2 \leq N \leq 9$
0.1%	$0 \leq N \leq 1$	$0 \leq N \leq 2$	$0 \leq N \leq 3$	$0 \leq N \leq 3$
0.01%	$0 \leq N \leq 0$	$0 \leq N \leq 0$	$0 \leq N \leq 1$	$0 \leq N \leq 1$

Table 1: No rejection regions for a 5% test size.

3.2 Conditional coverage

A more complete test was made by Christoffersen (1998), who developed a likelihood ratio statistic to test the joint assumption of unconditional coverage and independence of failures. Its main advantage over the previous statistic is that it takes account of any conditionality in our forecast: if volatilities are low in some period and high in others, the forecast should respond to this clustering event. The Christoffersen procedure enables us to separate clustering effects from distributional assumption effects. His statistic is computed as:

$$-2 \ln[(1 - p)^{T-N} p^N] + 2 \ln[(1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}] \sim \chi_2^2, \quad (13)$$

where n_{ij} is the number of observations with value i followed by j for $i, j = 0, 1$ and

$$\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$$

are the corresponding probabilities. If the sequence of I_t is independent, then the probabilities to observe or not observe a VaR violation in the next period must be equal, which can be written more formally as $\pi_{01} = \pi_{11} = p$. The main advantage of this test is that it can reject a VaR model that generates either too many or too few clustered violations, although it needs several hundred observations in order the test to be accurate.

4 Data and Results

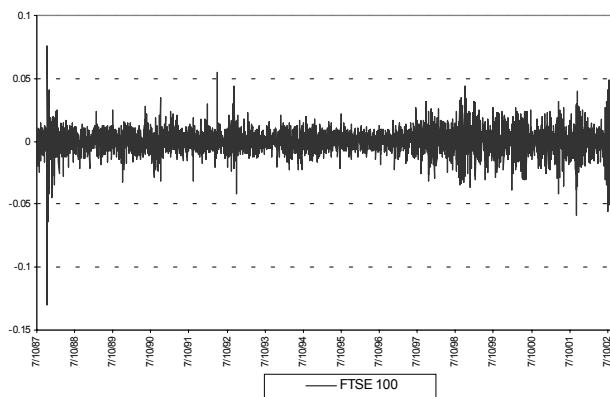
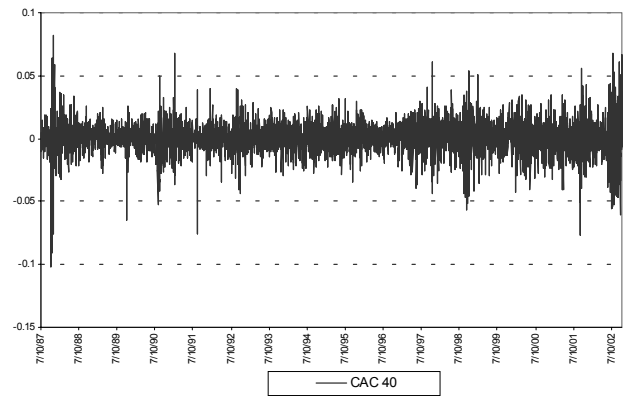
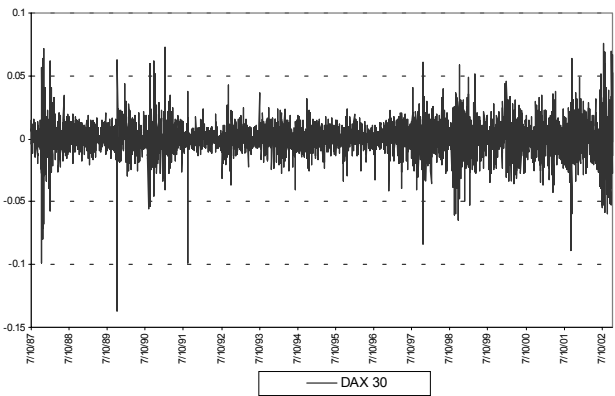
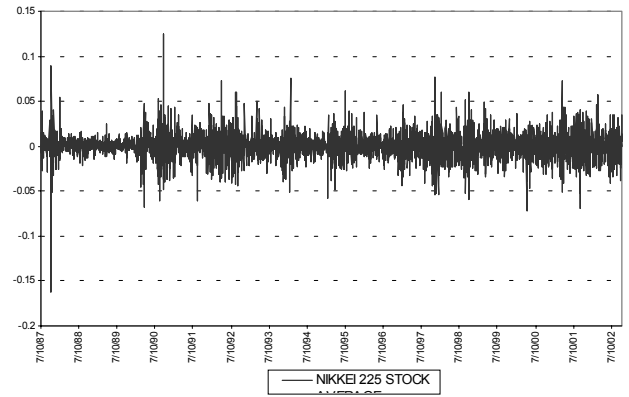
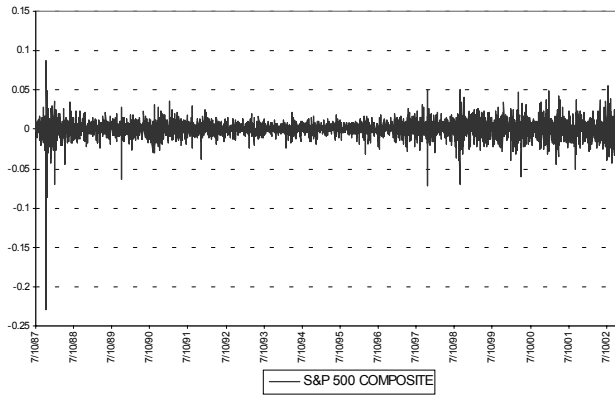
In order to evaluate the models, we generate out-of-sample VaR forecasts for five equity indices (CAC40, DAX30, FTSE100, NIKKEI225 and S&P500), obtained from DataStream for the period of July 9th, 1987 to October 18th, 2002. For all indices, we compute daily log returns and plot them. Volatility clustering is clearly visible in Figure(1).

Table 2 provides summary statistics as well as the Jarque–Bera statistic for testing normality. In all cases, the null hypothesis of normality is rejected at any level of significance, as there is evidence of significant excess kurtosis and negative skewness.

	S&P 500	NIKKEI 225	DAX 30	CAC 40	FTSE 100
Mean	0.00027	-0.00026	0.00021	0.00020	0.00014
Median	0.00042	-0.00011	0.00079	0.00034	0.00048
Maximum	0.08709	0.12430	0.07553	0.08225	0.07597
Minimum	-0.22833	-0.16135	-0.13710	-0.10138	-0.13029
Std. Deviation	0.01140	0.01494	0.01455	0.01377	0.01091
Skewness	-2.28112	-0.05997	-0.57695	-0.32182	-0.87756
Kurtosis	49.28424	10.10980	9.78510	7.39398	14.39124
Jarque - Bera	347258.8	7936.4	7569.2	3143.9	21348.6
Probability	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2: Descriptive statistics for the five equity indices

Figure 1. Continuously Compounded Daily Returns of Equity Indices (S&P500, NIKKEI225, DAX30, CAC40 and FTSE100) from July 9th, 1987 to October 18th, 2002



We will first use the most common parsimonious ARCH process i.e., the AR(1)-GARCH(1,1) model given by:

$$y_t = c_0 + c_1 y_{t-1} + z_t \sigma_t \quad (14)$$

$$z_t \stackrel{IID}{\sim} D(0, 1) \quad (15)$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 \quad (16)$$

assuming three different density functions for z_t : the normal, the Student-t and the Generalized Error Distribution as presented in section 2. The models are estimated using the entire dataset available and results are presented in Table 3. We note, first, that the conditional variance parameters are highly significant; second, that the distribution of the z_t is significantly thicker tailed than the normal distribution and, finally, that the parameter c_1 , which allows for the autocorrelation induced by discontinuous trading, is not significant in all cases.

In what follows, we model the conditional mean as a k -th order autoregressive process and the conditional variance as a GARCH, EGARCH or TARARCH process. Using equations (7, 8, 9) presented in Section 2, we will first apply ARCH processes to a GARCH, an EGARCH and a TARARCH model, all with parameters (p, q) and assuming a normal underlying distribution. We apply these models to the data using parameter values of $k = 0, \dots, 4$, $p = 0, 1, 2$ and $q = 1, 2$, yielding a total of 85 models². Moreover, under the additional assumptions of standardized residuals following a thicker tailed distribution (Student and GED), we reapply the AR(1) model to a GARCH(p, q), EGARCH(p, q) and TARARCH(p, q) framework, yielding another 34 models. Cases, for which the numerical maximization of the log-likelihood function failed to converge more than four times³, were excluded. On the other hand, for the models that the non-converged cases were less than four, the VaR forecasts were computed based on previous trading day parameter estimation.

For all models and all equity indices, we used a rolling sample of 500, 1000, 1500 and 2000 observations with the same number of VaR forecasts for each sample size. We generated one-day VaR forecasts both for 95% and 99% confidence levels, as recommended by the Basel Committee. The parameters of the models are re-estimated every trading day and all tests are performed using the information that would have been available at the time the estimate was made.

Due to the enormous size of the results that have been generated, we will proceed as follows. First, we present the VaR forecasts estimated using the standard normal distribution assumption for all sample sizes and we will examine whether the performance of the models is invariant to the choice of the sample size. We will then go on to present the contributions of alternative distributions, namely the standardized Student-t and the generalized error distribution, to the VaR framework.

4.1 Normal Distribution

The assumption of normality produces very weak results: the vast majority of models, irrespective of sample size chosen, understate the true one-day 95% VaR estimate and the exception rates found are higher than the 5% level predicted. The performance of the models is heavily dependent on the

²Numerical maximization of the log-likelihood function, for the EGARCH(2,2) model, frequently failed to converge. The five EGARCH models for these parameters were, therefore, excluded.

³Four times were the maximum number that one model failed to converge under the normal distribution assumption.

Parameter	S&P 500	NIKKEI 225	DAX 30	CAC 40	FTSE 100
<i>Normal Distribution</i>					
c_0	0.000557 (0.000132)	0.000549 (0.000192)	0.000614 (0.000199)	0.000432 (0.000197)	0.000395 (0.000150)
c_1	0.028859 (0.017917)	0.015127 (0.019737)	0.037770 (0.019223)	0.047177 (0.017319)	0.058032 (0.017804)
a_0	1.64E-06 (5.53E-07)	4.43E-06 (1.68E-06)	6.76E-06 (3.07E-06)	5.57E-06 (1.95E-06)	3.22E-06 (1.01E-06)
a_1	0.103524 (0.037118)	0.159935 (0.049919)	0.136356 (0.035324)	0.105451 (0.018500)	0.116497 (0.026331)
b_1	0.890270 (0.030393)	0.838772 (0.040506)	0.837132 (0.033176)	0.865466 (0.023878)	0.859395 (0.026836)
Log Likelihood	12468.68	10913.64	11357.62	11397.68	12492.48
<i>Student-t Distribution</i>					
c_0	0.000594 (0.000121)	0.000275 (0.000171)	0.000711 (0.000163)	0.000531 (0.000178)	0.000409 (0.000135)
c_1	0.012507 (0.016490)	-0.006850 (0.016965)	0.017391 (0.017283)	0.042970 (0.017279)	0.049854 (0.017128)
a_0	8.77E-07 2.23E-07	2.07E-06 (5.11E-07)	2.61E-06 (6.01E-07)	3.70E-06 (8.47E-07)	2.21E-06 (4.60E-07)
a_1	0.070246 (0.008139)	0.103547 (0.011004)	0.103558 (0.011609)	0.086050 (0.010117)	0.087884 (0.010318)
b_1	0.925879 (0.007779)	0.895057 (0.009965)	0.886968 (0.011657)	0.894520 (0.012412)	0.891897 (0.011782)
v	5.60013 (0.429782)	6.097681 (0.473549)	7.947894 (0.547896)	10.49132 (1.124427)	11.07864 (0.927738)
Log Likelihood	12638.65	11076.65	11532.06	11445.73	12578.62
<i>Generalized Error Distribution</i>					
c_0	0.000560 (0.000116)	0.000306 (0.000166)	0.000733 (0.000163)	0.000454 (0.000178)	0.000432 (0.000135)
c_1	-0.000820 (0.015669)	-0.006770 (0.016497)	0.017969 (0.017162)	0.038205 (0.017206)	0.046718 (0.017111)
a_0	1.11E-06 (2.60E-07)	3.19E-06 (5.96E-07)	4.13E-06 (8.16E-07)	4.78E-06 (9.61E-07)	2.67E-06 (5.60E-07)
a_1	0.078919 (0.007099)	0.119683 (0.012156)	0.118569 (0.011676)	0.094767 (0.010579)	0.098021 (0.011616)
b_1	0.915449 (0.007706)	0.875434 (0.011438)	0.866873 (0.013691)	0.879949 (0.013724)	0.879750 (0.013507)
v	1.250571 (0.024604)	1.305231 (0.023113)	1.395110 (0.021377)	1.552809 (0.035018)	1.545649 (0.019470)
Log Likelihood	12616.52	11040.99	11471.80	11429.26	12541.29
Observations	3853	3767	3835	3826	3857

Table 3: Parameter estimates of the AR(1) GARCH(1,1) model for all five indices, using the entire dataset and assuming three different distributions for the standardized residuals. Standard errors are presented in parentheses.

stock index, with the Nikkei 225 index faring somewhat better than any other. The p-values for both conditional and unconditional coverage are relatively low for all indices, with the European showing the lowest probability values.

More specifically, we observe that, both at the 2000 and 1500 sample size, none out of the 85 models produces an exception rate below the predicted 5%. In very few cases, the estimated exception rate is close enough to the expected one (e.g. for the S&P 500 and the NIKKEI 225 indices, the AR(1) GARCH(1,1) and the AR(1) EGARCH(1,2) models achieve the closest fit, respectively). Moreover, in almost all cases, the different specification of the conditional mean or the introduction of only ARCH terms in the conditional variance does not improve significantly either the unconditional or the conditional coverage of the models. In the case of the S&P 500, the average p-value for the null hypothesis of unconditional coverage of the GARCH(1,1) family is 56%, while the best performing family for the NIKKEI 225 and the CAC 40 indices is the EGARCH(1,2) one, with corresponding average p-values equal to 79,1% and 13,6%, respectively. Similarly, for the DAX 30 and the FTSE 100 indices, we conclude that the null hypothesis of correct unconditional coverage is rejected for all the models at the 10% level of significance, while the hypothesis of conditional coverage is rejected at the 15% level. It is also interesting to point out that, although the EGARCH structure captures better the asymmetric relation between returns and volatility changes and produces better unconditional VaR predictions than the TARCH family, it does not produce more accurate 95% daily VaR forecasts than the much simpler and less flexible GARCH (p, q) structure.

As for the effect of the sample size, it seems that as the latter gets smaller, the results improve at all levels. This is due to the weight given to the latest volatility observations when using a smaller sample: the *smaller* sample size captures only the *latest* market movements and the produced VaR estimate is, hence, less dependent on the long run volatility trend, in the same sense that a 200-point moving average is smoother than a 50-point one. Consequently, the VaR estimate is less often rejected at the 95% confidence level, for all models and indices. Such behavioural trend is traced even at the 1000 observations sample, and becomes clearer at the 500 observations level, at least for three out of the five indices. The only exception to this “rule of thumb” is the S&P 500, where the highest p-values are achieved for the largest sample size.

For the 99% daily VaR estimates, the exception rates are all higher than the predicted 1% for all sample sizes, with the sample size having no effect at all in improving results. As for the coverage metrics, they are very low, ranging from 45.51% conditional coverage, for the EGARCH family on the Nikkei 225 index, down to 1.19% (!) for the same metric with the same modeling family applied to the S&P 500 index. Such performance, at least for the latter index, is suggestive of a “hump” in the tail distribution of returns: this means it seriously underestimates risk at a high level of confidence whereas it produces somewhat better results at the lower confidence level, yielding respectable p-values for the 95% VaR metric but extremely low ones for 99% one.

Table 4 shows the most appropriate models according to conditional and unconditional coverage, defined as the one with the highest p-value among all the estimated models for all sample sizes⁴. Both coverage measures suggest the same models in most of the cases. For all indices, p-values are deemed relatively low, with the weakest ones for the European indices: we believe this is due to the inadequate description of such data using the normal distribution. The normal distribution performs best with the Nikkei 225 index in both the 95% and the 99% level.

⁴In case of identical p-values being achieved by more than one model, only one of these models is given. In the appendix we present the results for all models and distribution assumptions.

Index	Unconditional Coverage			Conditional Coverage		
	<i>95% daily VaR forecasts</i>					
Index	Size	Model	Pr.	Size	Model	Pr.
S&P 500	2000	AR(1) GARCH(1,1)	72.25%	2000	AR(1) GARCH(1,1)	79.48%
NIKKEI 225	1000	AR(0) EGARCH(1,1)	96.95%	500	AR(3) EGARCH(1,2)	93.14%
DAX 30	500	AR(3) TARCH(1,2)	32.92%	500	AR(3) TARCH(1,2)	57.61%
CAC 40	1000	AR(1) EGARCH(1,2)	54.44%	1000	AR(1) EGARCH(1,2)	56.38%
FTSE 100	500	AR(0) EGARCH(2,1)	33.72%	500	AR(1) TARCH(2,1)	53.27%
<i>99% daily VaR forecasts</i>						
Index	Size	Model	Pr.	Size	Model	Pr.
S&P 500	1000	AR(0) EGARCH(1,2)	1.40%	1000	AR(0) EGARCH(1,2)	1.19%
NIKKEI 225	2000	AR(0) EGARCH(2,1)	31.87%	2000	AR(0) EGARCH(2,1)	45.51%
DAX 30	500	AR(3) GARCH(1,1)	13.92%	500	AR(3) GARCH(1,1)	23.40%
CAC 40	500	AR(2) GARCH(1,2)	8.70%	500	AR(2) GARCH(1,2)	15.60%
FTSE 100	1000	AR(0) EGARCH(1,1)	22.56%	1000	AR(0) EGARCH(1,1)	34.59%

Table 4: Optimal Models and Probability Values of coverage metrics for Normally distributed innovations

The main points of this thorough investigation of ARCH processes can be summarized in the following points. First of all, the GARCH term in the conditional variance plays an important role since it provides models with a longer memory and a more flexible lag structure. Secondly, the choice of sample size is important in the adequacy of the conditional variance forecasts. To our best knowledge, there is no persistent method in the literature for choosing the appropriate sample size. In most studies, researchers make an arbitrary choice of a rolling sample. Nevertheless, Engle et al. (1993) applied three different sample sizes of 300, 1000 and 5000 observations and noted, “that some restrictions on the length of forecasting sample may be profitable”. Our study reaches a similar result, as different sample sizes seem to be the most appropriate for different indices.

Generally speaking, the conditional mean specification is invariant to the VaR forecast accuracy, because the adequacy of the models does not depend on the autoregressive order. However, there are some cases where the mean specification offers an increasing precision in VaR predictions. An example worth noting is the AR(2) GARCH(1,2) model with a 500-point sample size for the CAC 40 index, which produces the most accurate forecasts for the 99% daily VaR, though the rest of the models of the GARCH(1,2) family do not produce adequate forecasts.

Lastly, the assumption of normally distributed standardized residuals provides us with adequate forecasts only at the 95% level. For each index, there is at least one family of ARCH processes, producing convenient predictions. On the other hand, normal distribution fails to produce useful estimations at the 99% level. Under the assumption that the residuals are conditionally normal distributed, their unconditional distribution has thicker tails than the normal one. However, the degree of leptokurtosis induced by the ARCH process does not often capture all of the leptokurtosis present in the data. In the next section, we introduce non-normality for the conditional distribution of the residuals in order to produce unconditional distribution with thicker tails.

4.2 Leptokurtic Distributions

As we have already pointed out in previous sections, the normal distribution assumption does not generate accurately one-step-ahead VaR numbers. In order to model more adequately the thickness of tails, we will use two different distributional assumptions for the standardized residuals: Student's-t and the Generalized Error Distribution (GED). In the previous section, we found out that the precision of forecasts does not depend on the various structures of the conditional mean. We will, therefore, apply only the AR(1) specification for the conditional mean, consistent with the non-synchronous trading effect. Models that failed to converge more than four times are totally excluded from the study.

Turning now to the distribution used for the innovations, it seems that the Student's-t is a better choice overall, based on Tables 5 and 6 which present the models with the highest p-values for the two leptokurtic distributions. Given the fat tails observed in most indices, the normal distribution fails prominently if no leptokurtosis is added implicitly to the model via a GARCH mechanism. The best results are produced for the Nikkei 225 index, where the normality assumption does not significantly increase exception rates, although, through the Jarque - Bera test, it is strictly rejected as a null hypothesis. We also observe it "works" better for the 95% VaR estimates, perhaps implying the existence of thicker tails at the lowest end (below 95% and above 99%) of the distribution. The GED behaves similarly to the normal for the 95% confidence level but yields better results at the higher 99% one: given that such a distribution exhibits thicker tails than the normal, it seems that the "hump" problem is here resolved. Its p-values for both conditional and unconditional coverage are, however, still in the 20-70% range for the S&P 500, DAX 30, CAC 40 and FTSE 100 indices. Using the Student's-t distribution, improves probability values, as they range from 80% to 95%, for all volatility models and all sample sizes, for these four indices. As for the Nikkei 225, it seems that the choice of this distribution overcorrects for thick tails and, consequently, either the Normal (at the 95% level), or the GED (at the 99% level), provide the better estimates.

For the 95% VaR level, under Student's-t assumption, there are evidences that GARCH or EGARCH model produce better forecasts than the corresponding TARARCH, while for the GED this is the case only for the GARCH family. By increasing the confidence level of VaR, the results are become more mixed because both symmetric and asymmetric models have been selected as optimal. The choice of the sample is turning out to be one of the most important factor in the VaR framework, because the selected model for one sample size is not optimal for the others, while there are extreme cases that produce totally contrary results. For example, the best performing model for the S&P500 index at the 95% VaR level is the AR(1) GARCH(0,2), when assuming the Student-t distribution and using a sample size equal to 2000 observations. Yet, had we used the 500 observations sample size, this model would have been characterized as one of the worst. This conclusion is common for all three distribution assumptions and both confidence levels, revealing the importance of the sample size.

Putting together a leptokurtic distribution, to capture fat tails, and a low complexity volatility model, to capture volatility clustering, yields the best combination, which was expected for the 99% VaR level because the parameter models the degree of kurtosis and has the ability to capture the extreme events. As far as the case of 95% VaR level, the excellent performance of the volatility models under the Student-t distribution was a surprise for us, as researchers always reported an overestimation of the risk (see Guermat and Harris (2002), Billio and Pelizzon (2000) for more

information). These results are summarized in Table 7, which demonstrates the optimal models and their corresponding p-values for all distribution assumptions.

	Unconditional Coverage				Conditional Coverage			
	<i>95% daily VaR forecasts</i>							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	1500	AR(1)	GARCH(0,2)	97.03%	1000	AR(1)	EGARCH(0,1)	93.39%
NIKKEI 225	500	AR(1)	EGARCH(1,2)	6.56%	500	AR(1)	EGARCH(1,2)	14.69%
DAX 30	500	AR(1)	EGARCH(0,2)	97.86%	1500	AR(1)	EGARCH(1,2)	93.21%
CAC 40	1500	AR(1)	EGARCH(2,1)	97.43%	1000	AR(1)	EGARCH(2,1)	93.25%
FTSE 100	1500	AR(1)	GARCH(1,2)	92.78%	1500	AR(1)	TARCH(1,1)	93.61%
<i>99% daily VaR forecasts</i>								
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	1000	AR(1)	EGARCH(2,1)	91.30%	1500	AR(1)	EGARCH(0,2)	82.35%
NIKKEI 225	500	AR(1)	EGARCH(2,1)	36.26%	500	AR(1)	EGARCH(2,1)	58.61%
DAX 30	1500	AR(1)	EGARCH(1,1)	93.43%	1000	AR(1)	EGARCH(1,1)	82.56%
CAC 40	1500	AR(1)	TARCH(1,1)	95.11%	1000	AR(1)	GARCH(1,2)	82.61%
FTSE 100	1000	AR(1)	TARCH(1,1)	92.04%	1000	AR(1)	TARCH(1,1)	80.91%

Table 5: Optimal Models and Probability Values of coverage metrics for t-distributed innovations

	Unconditional Coverage				Conditional Coverage			
	<i>95% daily VaR forecasts</i>							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	2000	AR(1)	GARCH(1,1)	72.25%	2000	AR(1)	GARCH(1,1)	79.48%
NIKKEI 225	500	AR(1)	GARCH(1,2)	61.47%	500	AR(1)	GARCH(1,2)	52.22%
DAX 30	500	AR(1)	GARCH(2,2)	72.92%	500	AR(1)	GARCH(2,2)	57.66%
CAC 40	1500	AR(1)	GARCH(2,1)	21.79%	1500	AR(1)	GARCH(2,1)	38.84%
FTSE 100	500	AR(1)	GARCH(2,2)	20.47%	500	AR(1)	GARCH(2,2)	38.15%
<i>99% daily VaR forecasts</i>								
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	2000	AR(1)	GARCH(1,1)	57.23%	2000	AR(1)	GARCH(1,1)	66.25%
NIKKEI 225	500	AR(1)	GARCH(1,2)	93.73%	1500	AR(1)	GARCH(0,1)	82.87%
DAX 30	2000	AR(1)	TARCH(1,1)	54.33%	2000	AR(1)	TARCH(1,1)	64.44%
CAC 40	2000	AR(1)	TARCH(1,1)	19.78%	2000	AR(1)	TARCH(1,1)	26.41%
FTSE 100	1500	AR(1)	TARCH(1,1)	57.88%	1500	AR(1)	TARCH(1,1)	66.64%

Table 6: Optimal Models and Probability Values of coverage metrics for GE-distributed innovations

5 Conclusion

Following the extensive and detailed investigation of a plethora of volatility modeling techniques, briefly presented in the preceding sections, a number of comments are of order, aiming to summarize

our results and give, both to the researcher and the practitioner, some fundamental guidelines with which to proceed in VaR estimation.

We have strong indications the mean process specification plays no important role here. Trying to extract autoregressive phenomena from the returns such that only the underlying volatility is left in the residuals, we experimented with a number of AR processes. Our results show that such a methodology does *not* add anything significant to the VaR framework other than complexity in the estimation procedure. Moreover, using only an ARCH term (without any lagged conditional variances) sometimes yields acceptable results only when residuals are modeled under either the Student's-t distribution or the GED; it is never the case for a Normal distribution. Generally speaking, in the VaR framework the leptokurtic distributions and especially the Student's-t, are more appropriate than the Normal assumption, as they generate more accurate conditional or unconditional forecasts, while there is no volatility model which is clearly superior than the others. Furthermore, the size of the rolling sample used in estimation turns out to be rather important: in simpler models and low confidence levels a sample size smaller than 2000 improves probability values. In more complex models, where leptokurtic distributions are used or where the confidence level chosen is high, a small sample size may lead to lack of convergence in the estimation algorithms. Finally, there is no consistent relation between the sample sizes and the optimal models, as we observe significant differences in the VaR forecasts for the same model under the four sample sizes.

		Unconditional Coverage				Conditional Coverage			
		<i>95% daily VaR forecasts</i>							
Index	Size	Distribution	Model	Pr.	Size	Distribution	Model	Pr.	
S&P500	2000	T	AR(1) GARCH(0,2)	97.03%	2000	T	AR(1) GARCH(0,1)	93.39%	
NIKKEI 225	1000	N	AR(0) EGARCH(1,1)	96.95%	500	N	AR(3) EGARCH(1,2)	93.14%	
DAX 30	500	T	AR(1) EGARCH(0,2)	97.86%	1500	T	AR(1) EGARCH(1,2)	93.21%	
CAC 40	1500	T	AR(1) EGARCH(1,2)	97.43%	1000	T	AR(1) EGARCH(2,1)	93.25%	
FTSE 100	1500	T	AR(1) GARCH(1,2)	92.78%	1500	T	AR(1) TARCH(1,1)	93.61%	
		<i>99% daily VaR forecasts</i>							
Index	Size	Distribution	Model	Pr.	Size	Distribution	Model	Pr.	
S&P 500	1000	T	AR(1) EGARCH(2,1)	91.30%	1500	T	AR(1) EGARCH(0,2)	82.35%	
NIKKEI 225	2000	G	AR(1) EGARCH(0,1)	93.73%	2000	G	AR(1) GARCH(0,1)	82.87%	
DAX 30	2000	T	AR(1) GARCH(0,2)	93.43%	2000	T	AR(1) GARCH(0,2)	82.56%	
CAC 40	1500	T	AR(1) TARCH(1,1)	95.11%	1000	T	AR(1) GARCH(1,2)	82.61%	
FTSE 100	1000	T	AR(1) TARCH(1,1)	92.04%	1000	T	AR(1) TARCH(1,1)	80.91%	

Table 7: Overall Optimal Models according to probability values of coverage metrics for Normal (N), Student (T) or GE (G) distribution of innovations.

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Appendix

Table (1.A). Normal Distribution, 2000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	8.96%	0.00%	0.00%	6.11%	3.79%	0.96%	8.28%	0.00%	0.00%	7.72%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(1)GARCH(0,1)	8.96%	0.00%	0.00%	6.11%	3.79%	0.96%	8.39%	0.00%	0.00%	7.72%	0.00%	0.00%	7.92%	0.00%	0.00%
AR(2)GARCH(0,1)	8.96%	0.00%	0.00%	6.23%	2.26%	0.51%	8.50%	0.00%	0.00%	7.67%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(3)GARCH(0,1)	9.01%	0.00%	0.00%	6.17%	2.94%	0.70%	8.45%	0.00%	0.00%	7.61%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(4)GARCH(0,1)	8.96%	0.00%	0.00%	6.17%	2.94%	0.70%	8.66%	0.00%	0.00%	7.56%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(0)GARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.86%	7.52%	0.00%	0.00%	7.34%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(1)GARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.86%	7.57%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(2)GARCH(0,2)	8.04%	0.00%	0.00%	5.94%	7.72%	0.49%	7.57%	0.00%	0.00%	7.28%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(3)GARCH(0,2)	8.20%	0.00%	0.00%	5.89%	9.62%	0.65%	7.57%	0.00%	0.00%	7.34%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(4)GARCH(0,2)	8.26%	0.00%	0.00%	5.89%	9.62%	0.65%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(0)GARCH(1,1)	5.40%	43.90%	56.23%	6.17%	2.94%	8.36%	5.99%	5.77%	15.04%	6.08%	4.05%	7.80%	6.35%	1.00%	0.24%
AR(1)GARCH(1,1)	5.18%	72.25%	79.48%	6.17%	2.94%	8.36%	6.05%	4.56%	12.18%	6.08%	4.05%	7.80%	6.14%	2.94%	0.80%
AR(2)GARCH(1,1)	5.23%	64.53%	74.32%	6.23%	2.26%	6.55%	6.10%	3.58%	9.75%	6.13%	3.16%	6.60%	6.19%	2.28%	0.72%
AR(3)GARCH(1,1)	5.34%	50.30%	62.51%	6.28%	1.72%	5.07%	6.10%	3.58%	9.75%	6.08%	4.05%	5.25%	6.19%	2.28%	0.72%
AR(4)GARCH(1,1)	5.29%	57.20%	44.47%	6.23%	2.26%	6.55%	5.99%	5.77%	15.04%	6.24%	1.87%	3.26%	6.30%	1.33%	0.56%
AR(0)GARCH(1,2)	5.45%	38.00%	49.95%	6.17%	2.94%	6.66%	5.99%	5.77%	13.23%	6.02%	5.14%	6.00%	5.98%	6.05%	1.00%
AR(1)GARCH(1,2)	5.29%	57.20%	44.47%	6.23%	2.26%	5.11%	6.05%	4.56%	11.17%	6.08%	4.05%	5.25%	6.25%	1.75%	0.30%
AR(2)GARCH(1,2)	5.40%	43.90%	56.23%	6.23%	2.26%	5.11%	6.16%	2.78%	7.67%	6.13%	3.16%	4.53%	6.09%	3.77%	0.87%
AR(3)GARCH(1,2)	5.18%	72.25%	79.48%	6.17%	2.94%	6.66%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.19%	2.28%	1.40%
AR(4)GARCH(1,2)	5.29%	57.20%	44.47%	6.23%	2.26%	5.11%	6.16%	2.78%	7.67%	6.35%	1.08%	2.23%	6.14%	2.94%	1.58%
AR(0)GARCH(2,1)	5.61%	23.50%	18.48%	6.11%	3.79%	5.59%	6.21%	2.14%	6.65%	5.97%	6.48%	10.49%	6.14%	2.94%	1.58%
AR(1)GARCH(2,1)	5.23%	64.53%	49.27%	6.11%	3.79%	5.59%	6.27%	1.64%	5.02%	6.13%	3.16%	6.60%	6.09%	3.77%	1.77%
AR(2)GARCH(2,1)	5.40%	43.90%	34.87%	6.17%	2.94%	4.25%	6.32%	1.24%	4.08%	6.13%	3.16%	6.60%	6.19%	2.28%	1.40%
AR(3)GARCH(2,1)	5.50%	32.64%	57.55%	6.23%	2.26%	5.11%	6.10%	3.58%	9.75%	6.24%	1.87%	3.26%	6.09%	3.77%	1.77%
AR(4)GARCH(2,1)	5.40%	43.90%	56.23%	6.17%	2.94%	6.66%	6.21%	2.14%	6.65%	6.24%	1.87%	3.26%	6.14%	2.94%	1.58%
AR(0)GARCH(2,2)	5.72%	16.38%	35.74%	6.23%	2.26%	6.55%	6.10%	3.58%	10.34%	6.19%	2.44%	3.87%	6.14%	2.94%	0.80%
AR(1)GARCH(2,2)	5.61%	23.50%	46.52%	6.23%	2.26%	5.11%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.19%	2.28%	0.72%
AR(2)GARCH(2,2)	5.61%	23.50%	43.44%	6.28%	1.72%	3.87%	6.21%	2.14%	6.24%	6.24%	1.87%	3.26%	6.30%	1.33%	0.27%
AR(3)GARCH(2,2)	5.50%	32.64%	43.82%	6.17%	2.94%	6.66%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.30%	1.33%	0.27%
AR(4)GARCH(2,2)	5.56%	27.81%	52.07%	6.17%	2.94%	6.66%	6.16%	2.78%	8.34%	6.13%	3.16%	4.53%	6.25%	1.75%	0.64%
AR(0)TARCH(0,1)	9.07%	0.00%	0.00%	6.23%	2.26%	0.10%	8.28%	0.00%	0.00%	7.45%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(1)TARCH(0,1)	9.07%	0.00%	0.00%	6.23%	2.26%	0.10%	8.28%	0.00%	0.00%	7.45%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(2)TARCH(0,1)	9.07%	0.00%	0.00%	6.11%	3.79%	0.19%	8.23%	0.00%	0.00%	7.45%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(3)TARCH(0,1)	8.96%	0.00%	0.00%	6.17%	2.94%	0.14%	8.23%	0.00%	0.00%	7.50%	0.00%	0.00%	8.08%	0.00%	0.00%
AR(4)TARCH(0,1)	9.07%	0.00%	0.00%	6.17%	2.94%	0.14%	8.28%	0.00%	0.00%	7.50%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(0)TARCH(0,2)	8.20%	0.00%	0.00%	5.89%	9.62%	0.04%	7.41%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(1)TARCH(0,2)	8.26%	0.00%	0.00%	5.89%	9.62%	0.04%	7.47%	0.00%	0.00%	7.23%	0.00%	0.02%	7.86%	0.00%	0.00%
AR(2)TARCH(0,2)	8.20%	0.00%	0.00%	5.83%	11.88%	0.05%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(3)TARCH(0,2)	8.04%	0.00%	0.00%	5.83%	11.88%	0.05%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(4)TARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.05%	7.47%	0.00%	0.00%	7.28%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(0)TARCH(1,1)	6.15%	2.78%	5.81%	5.83%	11.88%	10.06%	5.94%	7.24%	10.16%	5.81%	12.34%	27.00%	5.98%	6.05%	10.47%
AR(1)TARCH(1,1)	6.15%	2.78%	5.81%	5.77%	14.55%	12.56%	5.94%	7.24%	10.16%	5.91%	8.10%	19.88%	6.14%	2.94%	4.56%
AR(2)TARCH(1,1)	6.21%	2.15%	4.46%	5.72%	17.66%	15.48%	5.99%	5.77%	8.00%	5.91%	8.10%	19.88%	6.14%	2.94%	4.56%
AR(3)TARCH(1,1)	6.04%	4.55%	6.06%	5.72%	17.66%	15.48%	6.05%	4.56%	6.22%	5.91%	8.10%	19.88%	6.19%	2.28%	3.87%
AR(4)TARCH(1,1)	6.10%	3.57%	7.49%	5.72%	17.66%	15.48%	5.99%	5.77%	8.00%	5.86%	10.04%	23.31%	6.19%	2.28%	3.87%
AR(0)TARCH(1,2)	6.15%	2.78%	3.54%	5.83%	11.88%	3.87%	5.99%	5.77%	15.31%	5.86%	10.04%	19.03%	5.92%	7.56%	7.74%
AR(1)TARCH(1,2)	5.88%	8.96%	6.38%	5.72%	17.66%	6.21%	6.10%	3.58%	10.34%	6.02%	5.14%	12.11%	6.14%	2.94%	2.84%
AR(2)TARCH(1,2)	6.04%	4.55%	6.06%	5.89%	9.62%	7.97%	6.05%	4.56%	12.66%	5.91%	8.10%	16.58%	6.03%	4.80%	6.09%
AR(3)TARCH(1,2)	6.04%	4.55%	6.06%	5.72%	17.66%	15.48%	5.99%	5.77%	15.31%	6.02%	5.14%	12.11%	6.03%	4.80%	6.09%
AR(4)TARCH(1,2)	6.15%	2.78%	1.65%	5.83%	11.88%	10.06%	6.10%	3.58%	10.34%	5.91%	8.10%	16.58%	6.09%	3.77%	5.30%
AR(0)TARCH(2,1)	6.04%	4.55%	6.06%	5.83%	11.88%	3.87%	6.05%	4.56%	9.72%	6.02%	5.14%	9.10%	6.09%	3.77%	7.70%
AR(1)TARCH(2,1)	5.94%	7.21%	9.90%	5.83%	11.88%	3.87%	6.05%	4.56%	9.72%	5.86%	10.04%	23.31%	6.03%	4.80%	9.04%
AR(2)TARCH(2,1)	5.88%	8.96%	12.43%	5.83%	11.88%	3.87%	6.10%	3.58%	4.78%	5.75%	15.04%	33.50%	5.98%	6.05%	10.47%
AR(3)TARCH(2,1)	6.10%	3.57%	4.66%	5.83%	11.88%	3.87%	6.16%	2.78%	5.91%	5.86%	10.04%	23.31%	6.09%	3.77%	5.30%
AR(4)TARCH(2,1)	5.94%	7.21%	9.90%	5.89%	9.62%	3.00%	6.05%	4.56%	6.22%	5.86%	10.04%	23.31%	6.19%	2.28%	5.40%
AR(0)TARCH(2,2)	5.94%	7.21%	4.98%	5.94%	7.72%	2.29%	5.94%	7.24%	15.25%	5.81%	12.34%	28.68%	5.98%	6.05%	10.47%
AR(1)TARCH(2,2)	5.88%	8.96%	6.38%	5.89%	9.62%	3.00%	6.10%	3.58%	9.75%	5.81%	12.34%	28.68%	5.98%	6.05%	10.47%
AR(2)TARCH(2,2)	5.94%	7.21%	4.98%	5.72%	17.66%	6.21%	5.99%	5.77%	15.04%	5.70%	18.18%	38.65%	5.98%	6.05%	10.47%
AR(3)TARCH(2,2)	5.88%	8.96%	6.38%	5.89%	9.62%	3.00%	6.10%	3.58%	9.75%	5.86%	10.04%	24.24%	5.98%	6.05%	6.91%
AR(4)TARCH(2,2)	5.99%	5.75%	3.84%	5.72%	17.66%	6.21%	6.05%	4.56%	12.18%	5.91%	8.10%	19.88%	6.03%	4.80%	6.09%
AR(0)EGARCH(0,1)	9.23%	0.00%	0.00%	6.06%	4.85%	1.30%	8.50%	0.00%	0.00%	7.56%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(1)EGARCH(0,1)	9.28%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.45%	0.00%	0.00%	8.40%	0.00%	0.00%
AR(2)EGARCH(0,1)	9.28%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(3)EGARCH(0,1)	9.34%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(4)EGARCH(0,1)	9.39%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(0)EGARCH(0,2)	8.80%	0.00%	0.00%	6.06%	4.85%	0.27%	8.12%	0.00%	0.00%	7.45%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.85%	0.00%	0.00%	6.11%	3.79%	0.19%	8.01%	0.00%	0.00%	7.45%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(2)EGARCH(0,2)	8.85%	0.00%	0.00%	6.11%	3.79%	0.01%	8.07%	0.00%	0.00%	7.56%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(3)EGARCH(0,2)	8.74%	0.00%	0.00%	6.06%	4.85%	0.01%	8.12%	0.00%	0.00%	7.50%	0.00%	0.00%	8.08%	0.00%	0.00%
AR(4)EGARCH(0,2)	8.69%	0.00%	0.00%	6.11%	3.79%	0.01%	8.07%	0.00%	0.00%	7.39%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(0)EGARCH(1,1)	5.56%	27.81%	49.69%	5.21%	69.22%	57.31%	6.16%	2.78%	7.71%	5.91%	8.10%	16.58%	6.30%	1.33%	1.78%
AR(1)EGARCH(1,1)	5.50%	32.64%	43.82%	5.21%	69.22%	57.31%	6.10%	3.58%	9.75%	5.86%	10.04%	19.03%	6.41%	0.75%	1.23%
AR(2)EGARCH(1,1)	5.61%	23.50%	43.44%	5.21%	69.22%	57.31%	6.32%	1.24%	3.98%	5.86%	10.04%	19.03%	6.41%	0.75%	1.23%
AR(3)EGARCH(1,1)	5.72%	16.38%	31.98%	5.15%	77.34%	62.05%	6.10%	3.58%	9.75%	5.91%	8.10%	11.94%	6.41%	0.75%	1.23%
AR(4)EGARCH(1,1)	5.56%	27.81%	37.96%	5.21%	69.22%	57.31%	6.10%	3.58%	9.75%	5.91%	8.10%	11.94%	6.35%	1.00%	1.49%
AR(0)EGARCH(1,2)	5.61%	23.50%	18.48%	5.15%	77.34%	62.05%	6.32%	1.24%	3.56%	5.81%	12.34%	14.88%	6.30%	1.33%	1.78%
AR(1)EGARCH(1,2)	5.61%	23.50%	18.48%	5.09%	85.75%	66.26%	6.21%	2.14%	6.03						

Table (2.A). Normal Distribution, 1500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	8.31%	0.00%	0.00%	6.23%	2.26%	0.51%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.38%	0.00%	0.00%
AR(1)GARCH(0,1)	8.31%	0.00%	0.00%	6.23%	2.26%	0.51%	8.56%	0.00%	0.00%	7.06%	0.01%	0.06%	7.54%	0.00%	0.00%
AR(2)GARCH(0,1)	8.36%	0.00%	0.00%	6.40%	0.98%	0.18%	8.50%	0.00%	0.00%	7.12%	0.01%	0.04%	7.65%	0.00%	0.00%
AR(3)GARCH(0,1)	8.36%	0.00%	0.00%	6.28%	1.72%	0.36%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.54%	0.00%	0.00%
AR(4)GARCH(0,1)	8.42%	0.00%	0.00%	6.40%	0.98%	0.18%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.59%	0.00%	0.00%
AR(0)GARCH(0,2)	8.09%	0.00%	0.00%	6.28%	1.72%	0.36%	7.41%	0.00%	0.00%	7.06%	0.01%	0.04%	7.38%	0.00%	0.00%
AR(1)GARCH(0,2)	8.20%	0.00%	0.00%	6.17%	2.94%	0.70%	7.36%	0.00%	0.01%	7.23%	0.00%	0.01%	7.49%	0.00%	0.00%
AR(2)GARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.36%	0.00%	0.01%	7.23%	0.00%	0.01%	7.43%	0.00%	0.00%
AR(3)GARCH(0,2)	8.09%	0.00%	0.00%	6.06%	4.85%	1.30%	7.41%	0.00%	0.00%	7.34%	0.00%	0.01%	7.43%	0.00%	0.00%
AR(4)GARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.36%	0.00%	0.01%	7.39%	0.00%	0.00%	7.49%	0.00%	0.00%
AR(0)GARCH(1,1)	5.34%	50.30%	62.51%	5.89%	9.62%	23.55%	6.10%	3.58%	9.75%	5.86%	10.04%	13.42%	6.14%	2.94%	0.80%
AR(1)GARCH(1,1)	5.34%	50.30%	62.51%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.81%	12.34%	21.57%	6.14%	2.94%	0.80%
AR(2)GARCH(1,1)	5.45%	38.00%	49.95%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.91%	8.10%	7.54%	6.09%	3.77%	0.39%
AR(3)GARCH(1,1)	5.40%	43.90%	56.23%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.91%	8.10%	7.54%	6.09%	3.77%	0.87%
AR(4)GARCH(1,1)	5.40%	43.90%	56.23%	5.89%	9.62%	23.55%	6.16%	2.78%	7.71%	5.86%	10.04%	13.42%	6.19%	2.28%	0.14%
AR(0)GARCH(1,2)	5.40%	43.90%	68.94%	5.66%	21.26%	32.06%	6.05%	4.56%	12.18%	5.91%	8.10%	11.94%	5.92%	7.56%	0.44%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	5.60%	25.37%	37.71%	6.16%	2.78%	7.71%	5.91%	8.10%	11.94%	5.98%	6.05%	0.43%
AR(2)GARCH(1,2)	5.45%	38.00%	62.61%	5.60%	25.37%	37.71%	6.10%	3.58%	9.75%	5.86%	10.04%	8.29%	5.92%	7.56%	0.44%
AR(3)GARCH(1,2)	5.45%	38.00%	62.61%	5.66%	21.26%	32.06%	6.10%	3.58%	9.75%	6.02%	5.14%	6.00%	6.03%	4.80%	0.41%
AR(4)GARCH(1,2)	5.34%	50.30%	74.95%	5.60%	25.37%	37.71%	6.10%	3.58%	9.75%	6.08%	4.05%	3.13%	5.98%	6.05%	1.00%
AR(0)GARCH(2,1)	5.61%	23.50%	43.44%	5.77%	14.55%	22.32%	6.27%	1.64%	4.66%	5.86%	10.04%	13.42%	5.98%	6.05%	1.00%
AR(1)GARCH(2,1)	5.40%	43.90%	56.23%	5.77%	14.55%	22.32%	6.27%	1.64%	4.66%	5.91%	8.10%	11.94%	6.09%	3.77%	0.87%
AR(2)GARCH(2,1)	5.40%	43.90%	56.23%	5.77%	14.55%	12.56%	6.32%	1.24%	3.56%	5.91%	8.10%	11.94%	5.92%	7.56%	2.27%
AR(3)GARCH(2,1)	5.34%	50.30%	62.51%	5.72%	17.66%	15.48%	6.49%	0.52%	1.48%	6.02%	5.14%	3.50%	5.98%	6.05%	2.12%
AR(4)GARCH(2,1)	5.50%	32.64%	43.82%	5.72%	17.66%	26.92%	6.10%	3.58%	7.63%	5.97%	6.48%	6.77%	6.09%	3.77%	0.16%
AR(0)GARCH(2,2)	5.77%	13.51%	18.92%	5.77%	14.55%	30.16%	6.27%	1.64%	5.24%	5.81%	12.34%	14.88%	6.03%	4.80%	0.41%
AR(1)GARCH(2,2)	5.61%	23.50%	32.48%	5.60%	25.37%	37.71%	6.16%	2.78%	7.71%	5.75%	15.04%	16.29%	5.98%	6.05%	0.43%
AR(2)GARCH(2,2)	5.67%	19.70%	37.50%	5.72%	17.66%	26.92%	6.43%	0.70%	2.00%	5.86%	10.04%	8.29%	5.92%	7.56%	1.05%
AR(3)GARCH(2,2)	5.72%	16.38%	33.10%	5.72%	17.66%	35.59%	6.54%	0.38%	1.08%	6.02%	5.14%	6.00%	6.19%	2.28%	0.14%
AR(4)GARCH(2,2)	5.61%	23.50%	43.44%	5.72%	17.66%	26.92%	6.16%	2.78%	7.71%	6.02%	5.14%	6.00%	6.09%	3.77%	0.16%
AR(0)TARCH(0,1)	8.36%	0.00%	0.00%	6.34%	1.30%	0.26%	8.45%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(1)TARCH(0,1)	8.42%	0.00%	0.00%	6.34%	1.30%	0.26%	8.39%	0.00%	0.00%	7.17%	0.01%	0.03%	7.70%	0.00%	0.00%
AR(2)TARCH(0,1)	8.42%	0.00%	0.00%	6.40%	0.98%	0.18%	8.45%	0.00%	0.00%	7.23%	0.00%	0.02%	7.75%	0.00%	0.00%
AR(3)TARCH(0,1)	8.36%	0.00%	0.00%	6.34%	1.30%	0.26%	8.45%	0.00%	0.00%	7.17%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(4)TARCH(0,1)	8.47%	0.00%	0.00%	6.34%	1.30%	0.26%	8.50%	0.00%	0.00%	7.17%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(0)TARCH(0,2)	8.31%	0.00%	0.00%	6.17%	2.94%	0.70%	7.25%	0.00%	0.02%	7.12%	0.01%	0.04%	7.54%	0.00%	0.00%
AR(1)TARCH(0,2)	8.31%	0.00%	0.00%	6.17%	2.94%	0.70%	7.25%	0.00%	0.01%	7.28%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(2)TARCH(0,2)	8.26%	0.00%	0.00%	6.06%	4.85%	1.30%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.70%	0.00%	0.00%
AR(3)TARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.65%	0.00%	0.00%
AR(4)TARCH(0,2)	8.15%	0.00%	0.00%	6.11%	3.79%	0.96%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.59%	0.00%	0.00%
AR(0)TARCH(1,1)	6.31%	1.25%	3.53%	5.66%	21.26%	18.85%	5.89%	9.00%	12.76%	5.75%	15.04%	24.13%	6.03%	4.80%	3.64%
AR(1)TARCH(1,1)	6.31%	1.25%	3.53%	5.77%	14.55%	12.56%	5.89%	9.00%	12.76%	5.81%	12.34%	21.57%	6.14%	2.94%	2.84%
AR(2)TARCH(1,1)	6.37%	0.94%	2.67%	5.66%	21.26%	18.85%	5.89%	9.00%	12.76%	5.70%	18.18%	34.85%	6.46%	0.56%	1.43%
AR(3)TARCH(1,1)	6.53%	0.38%	1.07%	5.72%	17.66%	15.48%	5.89%	9.00%	12.76%	5.81%	12.34%	21.57%	6.25%	1.75%	4.44%
AR(4)TARCH(1,1)	6.37%	0.94%	2.67%	5.60%	25.37%	22.66%	5.89%	9.00%	12.76%	5.81%	12.34%	14.88%	6.19%	2.28%	3.87%
AR(0)TARCH(1,2)	6.04%	4.55%	9.55%	5.43%	40.99%	16.34%	5.94%	7.24%	15.25%	5.59%	25.90%	31.26%	6.03%	4.80%	6.09%
AR(1)TARCH(1,2)	6.04%	4.55%	2.93%	5.49%	35.23%	13.80%	5.94%	7.24%	15.25%	5.59%	25.90%	31.26%	5.98%	6.05%	10.47%
AR(2)TARCH(1,2)	6.26%	1.64%	1.98%	5.43%	40.99%	16.34%	5.94%	7.24%	15.25%	5.64%	21.79%	29.06%	6.19%	2.28%	2.46%
AR(3)TARCH(1,2)	6.21%	2.15%	2.67%	5.49%	35.23%	13.80%	5.94%	7.24%	15.25%	5.75%	15.04%	16.29%	5.87%	9.38%	8.56%
AR(4)TARCH(1,2)	6.26%	1.64%	1.98%	5.60%	25.37%	9.49%	5.89%	9.00%	18.76%	5.75%	15.04%	16.29%	5.92%	7.56%	4.43%
AR(0)TARCH(2,1)	6.26%	1.64%	1.98%	5.60%	25.37%	9.49%	5.78%	13.59%	27.39%	5.81%	12.34%	27.00%	6.09%	3.77%	5.30%
AR(1)TARCH(2,1)	6.37%	0.94%	1.88%	5.49%	35.23%	13.80%	5.89%	9.00%	12.76%	5.81%	12.34%	27.00%	5.98%	6.05%	10.47%
AR(2)TARCH(2,1)	6.37%	0.94%	1.88%	5.49%	35.23%	31.53%	5.94%	7.24%	10.16%	5.70%	18.18%	34.85%	6.03%	4.80%	6.09%
AR(3)TARCH(2,1)	6.26%	1.64%	3.38%	5.49%	35.23%	13.80%	6.05%	4.56%	6.22%	5.64%	21.79%	38.84%	6.03%	4.80%	9.04%
AR(4)TARCH(2,1)	6.31%	1.25%	2.53%	5.55%	30.02%	11.52%	6.05%	4.56%	9.72%	5.81%	12.34%	21.57%	5.98%	6.05%	6.91%
AR(0)TARCH(2,2)	6.21%	2.15%	1.22%	5.60%	25.37%	9.49%	5.94%	7.24%	15.25%	5.81%	12.34%	27.00%	5.98%	6.05%	4.04%
AR(1)TARCH(2,2)	6.26%	1.64%	0.89%	5.55%	30.02%	11.52%	6.10%	3.58%	7.63%	5.81%	12.34%	21.57%	6.03%	4.80%	3.64%
AR(2)TARCH(2,2)	6.21%	2.15%	1.22%	5.60%	25.37%	9.49%	5.99%	5.77%	12.25%	5.81%	12.34%	27.00%	6.03%	4.80%	3.64%
AR(3)TARCH(2,2)	6.31%	1.25%	0.64%	5.43%	40.99%	16.34%	5.99%	5.77%	12.25%	5.81%	12.34%	21.57%	6.03%	4.80%	3.64%
AR(4)TARCH(2,2)	6.42%	0.70%	0.32%	5.43%	40.99%	16.34%	6.05%	4.56%	9.72%	5.64%	21.79%	44.02%	6.03%	4.80%	3.64%
AR(0)EGARCH(0,1)	8.74%	0.00%	0.00%	6.11%	3.79%	0.96%	8.45%	0.00%	0.00%	7.28%	0.00%	0.00%	7.75%	0.00%	0.00%
AR(1)EGARCH(0,1)	8.74%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.17%	0.01%	0.01%	7.86%	0.00%	0.00%
AR(2)EGARCH(0,1)	8.74%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.28%	0.00%	0.01%	7.92%	0.00%	0.00%
AR(3)EGARCH(0,1)	8.80%	0.00%	0.00%	6.00%	6.14%	1.74%	8.50%	0.00%	0.00%	7.23%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(4)EGARCH(0,1)	8.74%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.23%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(0)EGARCH(0,2)	8.20%	0.00%	0.00%	6.00%	6.14%	0.36%	7.85%	0.00%	0.00%	7.12%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.09%	0.00%	0.00%	6.06%	4.85%	0.27%	7.79%	0.00%	0.00%	7.17%	0.01%	0.02%	7.65%	0.00%	0.00%
AR(2)EGARCH(0,2)	8.15%	0.00%	0.00%	6.11%	3.79%	0.19%	7.79%	0.00%	0.00%	7.23%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(3)EGARCH(0,2)	8.09%	0.00%	0.00%	6.00%	6.14%	0.36%	7.79%	0.00%	0.00%	7.23%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(4)EGARCH(0,2)	7.99%	0.00%	0.00%	6.06%	4.85%	0.27%	7.90%	0.00%	0.00%	7.23%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(0)EGARCH(1,1)	5.88%	8.96%	18.44%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.81%	12.34%	14.88%	5.87%	9.38%	13.53%
AR(1)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.43%	0.70%	2.00%	5.64%	21.79%	29.06%	6.19%	2.28%	2.46%
AR(2)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.70%	18.18%	26.65%	6.25%	1.75%	2.11%
AR(3)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.86%	10.04%	13.42%	6.25%	1.75%	2.11%
AR(4)EGARCH(1,1)	5.88%	8.96%	18.44%	5.15%	77.34%	62.05%	6.32%	1.24%	3.56%	5.86%	10.04%	13.42%	6.25%	1.75%	2.11%
AR(0)EGARCH(1,2)	5.94%	7.21%	4.98%	5.15%	77.34%	62.05%	6.05%	4.56%	9.72%	5.48%	35.72%	34.76%	6.03%	4.80%	3.64%
AR(1)EGARCH(1,2)	6.04%	4.55%	6.06%	5.15%	77.34%	62.05%	6.05%	4.56%							

Table (3.A). Normal Distribution, 1000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	7.66%	0.00%	0.00%	5.77%	14.55%	4.93%	7.47%	0.00%	0.00%	6.68%	0.17%	0.53%	6.89%	0.04%	0.04%
AR(1)GARCH(0,1)	7.61%	0.00%	0.00%	5.89%	9.62%	3.00%	7.52%	0.00%	0.00%	6.63%	0.23%	0.70%	7.05%	0.01%	0.01%
AR(2)GARCH(0,1)	7.61%	0.00%	0.00%	6.00%	6.14%	1.74%	7.47%	0.00%	0.00%	6.79%	0.08%	0.30%	6.95%	0.03%	0.02%
AR(3)GARCH(0,1)	7.72%	0.00%	0.00%	5.89%	9.62%	3.00%	7.47%	0.00%	0.00%	6.85%	0.06%	0.22%	7.00%	0.02%	0.01%
AR(4)GARCH(0,1)	7.72%	0.00%	0.00%	6.00%	6.14%	1.74%	7.52%	0.00%	0.00%	6.85%	0.06%	0.22%	6.89%	0.04%	0.04%
AR(0)GARCH(0,2)	7.34%	0.00%	0.00%	5.83%	11.88%	10.06%	6.70%	0.14%	0.36%	6.35%	1.08%	2.99%	6.89%	0.04%	0.02%
AR(1)GARCH(0,2)	7.39%	0.00%	0.00%	5.89%	9.62%	7.97%	6.65%	0.20%	0.46%	6.35%	1.08%	2.99%	6.89%	0.04%	0.02%
AR(2)GARCH(0,2)	7.34%	0.00%	0.00%	5.83%	11.88%	3.87%	6.70%	0.14%	0.36%	6.41%	0.81%	1.22%	7.00%	0.02%	0.01%
AR(3)GARCH(0,2)	7.23%	0.00%	0.01%	5.77%	14.55%	4.93%	6.70%	0.14%	0.36%	6.46%	0.60%	1.45%	6.84%	0.06%	0.03%
AR(4)GARCH(0,2)	7.23%	0.00%	0.01%	6.06%	4.85%	3.67%	6.76%	0.10%	0.27%	6.35%	1.08%	2.23%	6.84%	0.06%	0.03%
AR(0)GARCH(1,1)	5.45%	38.00%	62.61%	5.77%	14.55%	22.32%	5.99%	5.77%	15.04%	5.81%	12.34%	21.57%	5.87%	9.38%	8.56%
AR(1)GARCH(1,1)	5.45%	38.00%	62.61%	5.77%	14.55%	22.32%	5.99%	5.77%	12.25%	6.02%	5.14%	12.11%	6.03%	4.80%	3.64%
AR(2)GARCH(1,1)	5.40%	43.90%	68.94%	5.83%	11.88%	25.23%	6.05%	4.56%	9.72%	5.64%	21.79%	29.06%	6.03%	4.80%	6.09%
AR(3)GARCH(1,1)	5.40%	43.90%	68.94%	5.83%	11.88%	25.23%	5.99%	5.77%	12.25%	5.81%	12.34%	14.88%	6.09%	3.77%	5.30%
AR(4)GARCH(1,1)	5.34%	50.30%	74.95%	5.83%	11.88%	25.23%	5.99%	5.77%	12.25%	5.81%	12.34%	14.88%	6.19%	2.28%	1.40%
AR(0)GARCH(1,2)	5.40%	43.90%	68.94%	6.00%	6.14%	13.70%	5.83%	11.11%	26.32%	5.70%	18.18%	17.60%	5.98%	6.05%	4.04%
AR(1)GARCH(1,2)	5.40%	43.90%	68.94%	6.00%	6.14%	13.70%	5.89%	9.00%	21.59%	5.91%	8.10%	11.94%	5.87%	9.38%	4.80%
AR(2)GARCH(1,2)	5.45%	38.00%	62.61%	6.00%	6.14%	16.15%	5.89%	9.00%	22.11%	5.75%	15.04%	16.29%	5.92%	7.56%	7.74%
AR(3)GARCH(1,2)	5.56%	27.81%	45.19%	6.00%	6.14%	16.15%	5.89%	9.00%	21.59%	5.86%	10.04%	13.42%	5.92%	7.56%	7.74%
AR(4)GARCH(1,2)	5.56%	27.81%	49.69%	5.94%	7.72%	17.00%	5.94%	7.24%	18.29%	5.86%	10.04%	13.42%	5.87%	9.38%	4.80%
AR(0)GARCH(2,1)	5.56%	27.81%	49.69%	5.66%	21.26%	18.85%	5.89%	9.00%	18.76%	5.97%	6.48%	14.26%	5.98%	6.05%	10.47%
AR(1)GARCH(2,1)	5.45%	38.00%	62.61%	5.66%	21.26%	7.72%	5.83%	11.11%	22.81%	5.91%	8.10%	16.58%	6.14%	2.94%	4.56%
AR(2)GARCH(2,1)	5.40%	43.90%	68.94%	5.77%	14.55%	22.32%	5.89%	9.00%	18.76%	6.02%	5.14%	9.10%	5.92%	7.56%	11.97%
AR(3)GARCH(2,1)	5.40%	43.90%	68.94%	5.77%	14.55%	22.32%	5.94%	7.24%	15.25%	5.81%	12.34%	14.88%	6.09%	3.77%	5.30%
AR(4)GARCH(2,1)	5.40%	43.90%	68.94%	5.66%	21.26%	18.85%	5.99%	5.77%	12.25%	6.02%	5.14%	9.10%	6.14%	2.94%	4.56%
AR(0)GARCH(2,2)	5.94%	7.21%	18.32%	5.94%	7.72%	17.00%	5.89%	9.00%	21.59%	5.64%	21.79%	18.77%	5.82%	11.54%	9.35%
AR(1)GARCH(2,2)	6.04%	4.55%	12.66%	5.89%	9.62%	20.84%	5.89%	9.00%	22.11%	5.59%	25.90%	19.74%	5.98%	6.05%	6.91%
AR(2)GARCH(2,2)	5.99%	5.75%	15.32%	5.89%	9.62%	20.84%	5.94%	7.24%	18.29%	5.70%	18.18%	17.60%	5.87%	9.38%	8.56%
AR(3)GARCH(2,2)	5.94%	7.21%	15.67%	5.83%	11.88%	25.23%	5.94%	7.24%	18.29%	5.81%	12.34%	21.57%	6.03%	4.80%	1.95%
AR(4)GARCH(2,2)	5.88%	8.96%	21.64%	5.89%	9.62%	20.84%	5.89%	9.00%	18.76%	5.97%	6.48%	14.26%	6.14%	2.94%	1.58%
AR(0)TARCH(0,1)	7.77%	0.00%	0.00%	5.77%	14.55%	1.12%	7.68%	0.00%	0.00%	6.63%	0.23%	0.85%	7.16%	0.01%	0.01%
AR(1)TARCH(0,1)	7.83%	0.00%	0.00%	5.77%	14.55%	1.12%	7.74%	0.00%	0.00%	6.68%	0.17%	0.64%	7.22%	0.00%	0.01%
AR(2)TARCH(0,1)	7.93%	0.00%	0.00%	5.94%	7.72%	2.29%	7.85%	0.00%	0.00%	6.63%	0.23%	0.85%	7.16%	0.01%	0.01%
AR(3)TARCH(0,1)	7.83%	0.00%	0.00%	5.83%	11.88%	3.87%	7.79%	0.00%	0.00%	6.96%	0.03%	0.13%	7.22%	0.00%	0.01%
AR(4)TARCH(0,1)	7.88%	0.00%	0.00%	5.94%	7.72%	2.29%	7.79%	0.00%	0.00%	6.79%	0.08%	0.35%	7.22%	0.00%	0.01%
AR(0)TARCH(0,2)	7.50%	0.00%	0.00%	5.66%	21.26%	7.72%	6.54%	0.38%	1.41%	6.35%	1.08%	2.99%	7.05%	0.01%	0.02%
AR(1)TARCH(0,2)	7.50%	0.00%	0.00%	5.66%	21.26%	7.72%	6.54%	0.38%	1.41%	6.46%	0.60%	1.87%	7.11%	0.01%	0.01%
AR(2)TARCH(0,2)	7.61%	0.00%	0.00%	5.77%	14.55%	4.93%	6.65%	0.20%	0.74%	6.41%	0.81%	1.81%	7.22%	0.00%	0.01%
AR(3)TARCH(0,2)	7.39%	0.00%	0.00%	5.77%	14.55%	4.93%	6.59%	0.28%	0.99%	6.41%	0.81%	2.38%	7.11%	0.01%	0.01%
AR(4)TARCH(0,2)	7.29%	0.00%	0.00%	5.89%	9.62%	3.00%	6.49%	0.52%	1.80%	6.35%	1.08%	2.99%	7.11%	0.01%	0.01%
AR(0)TARCH(1,1)	6.21%	2.15%	4.46%	5.32%	54.14%	22.01%	5.61%	23.69%	44.06%	5.70%	18.18%	26.65%	6.03%	4.80%	9.04%
AR(1)TARCH(1,1)	6.21%	2.15%	4.46%	5.43%	40.99%	36.48%	5.56%	28.04%	50.35%	5.75%	15.04%	16.29%	5.98%	6.05%	13.90%
AR(2)TARCH(1,1)	6.37%	0.94%	1.88%	5.32%	54.14%	46.95%	5.61%	23.69%	44.06%	5.64%	21.79%	29.06%	6.09%	3.77%	7.70%
AR(3)TARCH(1,1)	6.21%	2.15%	4.46%	5.26%	61.47%	52.22%	5.61%	23.69%	44.06%	5.70%	18.18%	26.65%	6.14%	2.94%	4.56%
AR(4)TARCH(1,1)	6.37%	0.94%	1.88%	5.26%	61.47%	52.22%	5.67%	19.85%	38.07%	5.91%	8.10%	11.94%	6.09%	3.77%	7.70%
AR(0)TARCH(1,2)	6.31%	1.25%	2.53%	5.38%	47.31%	19.08%	5.61%	23.69%	44.06%	5.75%	15.04%	24.13%	6.03%	4.80%	6.09%
AR(1)TARCH(1,2)	6.26%	1.64%	3.38%	5.49%	35.23%	31.53%	5.61%	23.69%	44.06%	5.64%	21.79%	18.77%	6.14%	2.94%	4.56%
AR(2)TARCH(1,2)	6.26%	1.64%	3.38%	5.38%	47.31%	41.66%	5.78%	13.59%	27.39%	5.86%	10.04%	13.42%	5.98%	6.05%	6.91%
AR(3)TARCH(1,2)	6.42%	0.70%	1.38%	5.43%	40.99%	36.48%	5.78%	13.59%	27.39%	5.81%	12.34%	14.88%	6.03%	4.80%	6.09%
AR(4)TARCH(1,2)	6.48%	0.52%	1.00%	5.43%	40.99%	36.48%	5.72%	16.49%	32.49%	5.86%	10.04%	13.42%	5.92%	7.56%	7.74%
AR(0)TARCH(2,1)	6.37%	0.94%	1.88%	5.60%	25.37%	9.49%	5.56%	28.04%	50.35%	5.70%	18.18%	26.65%	5.71%	17.05%	34.04%
AR(1)TARCH(2,1)	6.37%	0.94%	1.88%	5.32%	54.14%	22.01%	5.78%	13.59%	27.39%	5.75%	15.04%	24.13%	5.82%	11.54%	26.10%
AR(2)TARCH(2,1)	6.26%	1.64%	3.38%	5.43%	40.99%	16.34%	5.67%	19.85%	38.07%	5.70%	18.18%	34.85%	5.76%	14.09%	16.62%
AR(3)TARCH(2,1)	6.37%	0.94%	1.88%	5.38%	47.31%	19.08%	5.67%	19.85%	38.07%	5.59%	25.90%	42.72%	5.92%	7.56%	16.24%
AR(4)TARCH(2,1)	6.48%	0.52%	1.00%	5.38%	47.31%	19.08%	5.83%	11.11%	22.81%	5.86%	10.04%	19.03%	6.09%	3.77%	9.82%
AR(0)TARCH(2,2)	6.26%	1.64%	1.98%	5.43%	40.99%	36.48%	5.67%	19.85%	38.07%	5.64%	21.79%	38.84%	5.71%	17.05%	18.07%
AR(1)TARCH(2,2)	6.10%	3.57%	4.66%	5.43%	40.99%	36.48%	5.72%	16.49%	32.49%	5.81%	12.34%	21.57%	5.76%	14.09%	24.02%
AR(2)TARCH(2,2)	6.26%	1.64%	1.98%	5.43%	40.99%	16.34%	5.67%	19.85%	38.07%	5.91%	8.10%	11.94%	5.87%	9.38%	8.56%
AR(3)TARCH(2,2)	6.37%	0.94%	1.06%	5.26%	61.47%	25.06%	5.67%	19.85%	38.07%	5.97%	6.48%	10.49%	5.92%	7.56%	7.74%
AR(4)TARCH(2,2)	6.31%	1.25%	1.46%	5.49%	35.23%	13.80%	5.78%	13.59%	27.39%	5.91%	8.10%	11.94%	5.92%	7.56%	7.74%
AR(0)EGARCH(0,1)	7.93%	0.00%	0.00%	5.66%	21.26%	7.72%	7.63%	0.00%	0.00%	6.52%	0.44%	1.15%	7.22%	0.00%	0.00%
AR(1)EGARCH(0,1)	7.88%	0.00%	0.00%	5.72%	17.66%	6.21%	7.63%	0.00%	0.00%	6.57%	0.32%	0.90%	7.27%	0.00%	0.00%
AR(2)EGARCH(0,1)	7.93%	0.00%	0.00%	5.77%	14.55%	4.93%	7.63%	0.00%	0.00%	6.63%	0.23%	0.70%	7.27%	0.00%	0.00%
AR(3)EGARCH(0,1)	7.88%	0.00%	0.00%	5.89%	9.62%	3.00%	7.68%	0.00%	0.00%	6.74%	0.12%	0.40%	7.32%	0.00%	0.00%
AR(4)EGARCH(0,1)	7.88%	0.00%	0.00%	5.83%	11.88%	3.87%	7.68%	0.00%	0.00%	6.74%	0.12%	0.40%	7.32%	0.00%	0.00%
AR(0)EGARCH(0,2)	7.45%	0.00%	0.00%	5.89%	9.62%	7.97%	6.98%	0.02%	0.04%	6.30%	1.43%	2.71%	7.00%	0.02%	0.01%
AR(1)EGARCH(0,2)	7.56%	0.00%	0.00%	5.89%	9.62%	7.97%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.00%	0.02%	0.01%
AR(2)EGARCH(0,2)	7.56%	0.00%	0.00%	6.00%	6.14%	4.81%	6.81%	0.07%	0.09%	6.41%	0.81%	1.81%	7.00%	0.02%	0.01%
AR(3)EGARCH(0,2)	7.66%	0.00%	0.00%	5.94%	7.72%	6.23%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.05%	0.01%	0.01%
AR(4)EGARCH(0,2)	7.61%	0.00%	0.00%	6.06%	4.85%	3.67%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.11%	0.01%	0.00%
AR(0)EGARCH(1,1)	6.04%	4.55%	9.55%	4.98%	96.95%	72.50%	5.67%	19.85%	38.07%	5.48%	35.72%	34.76%	6.09%	3.77%	5.30%
AR(1)EGARCH(1,1)	6.04%	4.55%	9.55%	5.04%	94.35%	91.91%	5.67%	19.85%	38.07%	5.37%	47.69%	36.59%	6.14%	2.94%	2.84%
AR(2)EGARCH(1,1)	6.04%	4.55%	9.55%	5.21%	69.22%	81.16%	5.72%	16.49%	32.49%	5.48%	35.72%	20.97%	6.14%	2.94%	2.84%
AR(3)EGARCH(1,1)	6.04%	4.55%	9.55%	5.21%	69.22%	81.16%	5.72%	16.49%	32.49%	5.59%	25.90%	19.74%	6.14%	2.94%	2.84%
AR(4)EGARCH(1,1)	6.10%	3.57%	7.49%	5.38%	47.31%	63.30%	5.72%	16.49%	32.49%	5.64%	21.79%	18.77%	6.14%	2.94%	2.84%
AR(0)EGARCH(1,2)	5.83%	11.05%	8.08%	5.32%	54.14%	69.71%	5.99%	5.77%	15.04%	5.37%	47.69%	54.80%	5.92%	7.56%	7.74%
AR(1)EGARCH(1,2)															

Table (4.A). Normal Distribution, 500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	6.26%	1.64%	5.24%	5.83%	11.88%	3.87%	6.54%	0.38%	0.47%	6.52%	0.44%	1.15%	6.62%	0.22%	0.11%
AR(1)GARCH(0,1)	6.26%	1.64%	5.24%	5.89%	9.62%	7.97%	6.70%	0.14%	0.24%	6.57%	0.32%	0.65%	6.73%	0.11%	0.02%
AR(2)GARCH(0,1)	6.42%	0.70%	2.40%	6.00%	6.14%	9.31%	6.81%	0.07%	0.09%	6.52%	0.44%	1.15%	6.73%	0.11%	0.01%
AR(3)GARCH(0,1)	6.48%	0.52%	1.80%	6.00%	6.14%	9.31%	6.87%	0.05%	0.07%	6.63%	0.23%	0.51%	6.84%	0.06%	0.01%
AR(4)GARCH(0,1)	6.42%	0.70%	2.40%	6.00%	6.14%	9.31%	6.92%	0.03%	0.06%	6.63%	0.23%	0.51%	6.95%	0.03%	0.00%
AR(0)GARCH(0,2)	6.31%	1.25%	3.53%	5.66%	21.26%	18.55%	6.27%	1.64%	5.02%	6.30%	1.43%	2.71%	6.35%	1.00%	0.90%
AR(1)GARCH(0,2)	6.26%	1.64%	4.62%	5.77%	14.55%	12.56%	6.27%	1.64%	5.02%	6.24%	1.87%	3.26%	6.35%	1.00%	0.49%
AR(2)GARCH(0,2)	6.31%	1.25%	4.09%	5.72%	17.66%	15.48%	6.54%	0.38%	1.41%	6.24%	1.87%	3.26%	6.52%	0.41%	0.16%
AR(3)GARCH(0,2)	6.37%	0.94%	3.15%	5.77%	14.55%	12.56%	6.43%	0.70%	2.43%	6.30%	1.43%	2.71%	6.52%	0.41%	0.16%
AR(4)GARCH(0,2)	6.42%	0.70%	2.40%	5.89%	9.62%	7.97%	6.49%	0.52%	1.86%	6.35%	1.08%	2.23%	6.52%	0.41%	0.30%
AR(0)GARCH(1,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.05%	4.56%	12.66%	5.97%	6.48%	10.49%	5.71%	17.05%	26.68%
AR(1)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	5.99%	5.77%	15.31%	5.97%	6.48%	10.49%	5.76%	14.09%	24.02%
AR(2)GARCH(1,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.10%	3.58%	10.34%	5.64%	21.79%	18.77%	5.65%	20.47%	38.15%
AR(3)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	6.16%	2.78%	8.34%	5.91%	8.10%	7.54%	5.71%	17.05%	34.04%
AR(4)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	6.05%	4.56%	12.66%	5.86%	10.04%	13.42%	5.82%	11.54%	26.10%
AR(0)GARCH(1,2)	5.61%	23.50%	31.04%	5.72%	17.66%	26.92%	5.94%	7.24%	7.27%	5.81%	12.34%	9.00%	5.76%	14.09%	10.07%
AR(1)GARCH(1,2)	5.72%	16.38%	26.01%	5.77%	14.55%	22.32%	5.99%	5.77%	6.48%	5.81%	12.34%	9.00%	5.76%	14.09%	10.07%
AR(2)GARCH(1,2)	5.72%	16.38%	33.10%	5.72%	17.66%	26.92%	6.10%	3.58%	4.96%	5.91%	8.10%	11.94%	5.82%	11.54%	9.35%
AR(3)GARCH(1,2)	5.67%	19.70%	18.99%	5.66%	21.26%	32.06%	6.16%	2.78%	4.25%	5.64%	21.79%	18.77%	5.87%	9.38%	8.56%
AR(4)GARCH(1,2)	5.67%	19.70%	18.99%	5.60%	25.37%	37.71%	6.10%	3.58%	7.25%	5.75%	15.04%	16.29%	5.87%	9.38%	8.56%
AR(0)GARCH(2,1)	5.40%	43.90%	68.94%	5.49%	35.23%	31.53%	6.05%	4.56%	12.66%	5.86%	10.04%	8.29%	5.60%	24.37%	31.69%
AR(1)GARCH(2,1)	5.61%	23.50%	43.44%	5.43%	40.99%	36.48%	6.05%	4.56%	12.66%	5.86%	10.04%	23.31%	5.82%	11.54%	15.09%
AR(2)GARCH(2,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.27%	1.64%	5.24%	6.19%	2.44%	3.87%	5.76%	14.09%	29.99%
AR(3)GARCH(2,1)	5.67%	19.70%	41.03%	5.38%	47.31%	41.66%	6.21%	2.14%	6.65%	6.30%	1.43%	3.71%	5.82%	11.54%	26.10%
AR(4)GARCH(2,1)	5.50%	32.64%	56.13%	5.49%	35.23%	31.53%	6.10%	3.58%	10.34%	6.19%	2.44%	3.87%	5.87%	9.38%	22.43%
AR(0)GARCH(2,2)	5.61%	23.50%	31.04%	5.66%	21.26%	32.06%	5.83%	11.11%	20.43%	5.91%	8.10%	16.58%	5.60%	24.37%	42.22%
AR(1)GARCH(2,2)	5.72%	16.38%	26.01%	5.66%	21.26%	32.06%	5.78%	13.59%	23.02%	6.02%	5.14%	9.10%	5.98%	6.05%	1.00%
AR(2)GARCH(2,2)	5.77%	13.51%	29.09%	5.66%	21.26%	32.06%	6.16%	2.78%	6.10%	6.02%	5.14%	9.10%	5.98%	6.05%	6.91%
AR(3)GARCH(2,2)	5.77%	13.51%	16.20%	5.66%	21.26%	32.06%	6.10%	3.58%	4.96%	5.97%	6.48%	10.49%	6.09%	3.77%	3.23%
AR(4)GARCH(2,2)	5.83%	11.05%	14.67%	5.83%	11.88%	18.27%	6.10%	3.58%	7.25%	6.13%	3.16%	6.60%	5.82%	11.54%	15.09%
AR(0)TARCH(0,1)	6.69%	0.14%	0.25%	5.89%	9.62%	7.97%	6.70%	0.14%	0.56%	6.41%	0.81%	2.75%	6.73%	0.11%	0.13%
AR(1)TARCH(0,1)	6.64%	0.20%	0.36%	5.94%	7.72%	6.23%	6.81%	0.07%	0.30%	6.52%	0.44%	1.62%	6.84%	0.06%	0.08%
AR(2)TARCH(0,1)	6.64%	0.20%	0.36%	5.94%	7.72%	11.80%	6.87%	0.05%	0.19%	6.63%	0.23%	0.85%	6.89%	0.04%	0.04%
AR(3)TARCH(0,1)	6.75%	0.10%	0.17%	6.00%	6.14%	9.31%	6.87%	0.05%	0.19%	6.68%	0.17%	0.53%	7.00%	0.02%	0.02%
AR(4)TARCH(0,1)	6.64%	0.20%	0.36%	6.06%	4.85%	3.67%	6.81%	0.07%	0.26%	6.68%	0.17%	0.64%	7.00%	0.02%	0.02%
AR(0)TARCH(0,2)	6.69%	0.14%	0.12%	5.72%	17.66%	15.48%	6.16%	2.78%	8.34%	6.19%	2.44%	6.89%	6.52%	0.41%	0.81%
AR(1)TARCH(0,2)	6.42%	0.70%	0.76%	5.72%	17.66%	15.48%	6.32%	1.24%	4.08%	6.30%	1.43%	4.46%	6.62%	0.22%	0.51%
AR(2)TARCH(0,2)	6.48%	0.52%	0.54%	5.77%	14.55%	12.56%	6.32%	1.24%	4.08%	6.30%	1.43%	3.71%	6.79%	0.08%	0.16%
AR(3)TARCH(0,2)	6.53%	0.38%	0.71%	5.89%	9.62%	7.97%	6.43%	0.70%	2.39%	6.35%	1.08%	2.99%	6.73%	0.11%	0.21%
AR(4)TARCH(0,2)	6.64%	0.20%	0.36%	5.89%	9.62%	7.97%	6.54%	0.38%	1.34%	6.24%	1.87%	4.55%	6.79%	0.08%	0.16%
AR(0)TARCH(1,1)	5.77%	13.51%	26.95%	5.32%	54.14%	46.95%	5.67%	19.85%	38.07%	6.08%	4.05%	7.80%	5.55%	28.78%	46.12%
AR(1)TARCH(1,1)	5.77%	13.51%	26.95%	5.21%	69.22%	57.31%	5.78%	13.59%	27.39%	5.86%	10.04%	13.42%	5.71%	17.05%	34.04%
AR(2)TARCH(1,1)	5.61%	23.50%	43.44%	5.21%	69.22%	57.31%	5.94%	7.24%	15.25%	6.19%	2.44%	3.87%	5.60%	24.37%	31.69%
AR(3)TARCH(1,1)	5.77%	13.51%	26.95%	5.04%	94.35%	69.80%	5.78%	13.59%	27.39%	6.30%	1.43%	2.71%	5.82%	11.54%	21.35%
AR(4)TARCH(1,1)	5.94%	7.21%	18.20%	4.92%	88.26%	74.26%	5.83%	11.11%	22.81%	6.35%	1.08%	2.23%	5.76%	14.09%	29.99%
AR(0)TARCH(1,2)	5.94%	7.21%	18.20%	5.55%	30.02%	26.90%	5.61%	23.69%	40.96%	6.57%	0.32%	1.12%	5.87%	9.38%	13.53%
AR(1)TARCH(1,2)	5.88%	8.96%	21.94%	5.66%	21.26%	32.06%	5.67%	19.85%	41.22%	6.30%	1.43%	4.46%	5.71%	17.05%	26.68%
AR(2)TARCH(1,2)	5.94%	7.21%	18.20%	5.60%	25.37%	37.71%	5.56%	28.04%	52.20%	6.30%	1.43%	3.71%	5.65%	20.47%	19.39%
AR(3)TARCH(1,2)	5.83%	11.05%	26.13%	5.72%	17.66%	26.92%	5.50%	32.92%	57.61%	6.35%	1.08%	2.23%	5.87%	9.38%	13.53%
AR(4)TARCH(1,2)	5.94%	7.21%	18.20%	5.60%	25.37%	37.71%	5.67%	19.85%	41.22%	6.35%	1.08%	2.23%	5.76%	14.09%	24.02%
AR(0)TARCH(2,1)	5.72%	16.38%	35.74%	4.98%	96.95%	39.22%	5.78%	13.59%	27.39%	6.08%	4.05%	11.45%	5.55%	28.78%	33.86%
AR(1)TARCH(2,1)	5.99%	5.75%	14.91%	5.09%	85.75%	66.26%	5.78%	13.59%	27.39%	6.30%	1.43%	3.71%	5.55%	28.78%	53.27%
AR(2)TARCH(2,1)	5.94%	7.21%	18.20%	5.09%	85.75%	66.26%	5.94%	7.24%	15.25%	6.57%	0.32%	1.12%	5.82%	11.54%	20.96%
AR(3)TARCH(2,1)	5.99%	5.75%	14.91%	5.32%	54.14%	46.95%	5.89%	9.00%	22.11%	6.57%	0.32%	1.12%	5.82%	11.54%	21.35%
AR(4)TARCH(2,1)	6.04%	4.55%	12.66%	5.26%	61.47%	52.22%	5.83%	11.11%	22.81%	6.57%	0.32%	1.12%	5.87%	9.38%	13.53%
AR(0)TARCH(2,2)	5.72%	16.38%	31.98%	5.32%	54.14%	46.95%	5.72%	16.49%	35.94%	6.13%	3.16%	8.42%	5.65%	20.47%	29.27%
AR(1)TARCH(2,2)	5.94%	7.21%	14.99%	5.38%	47.31%	63.30%	5.83%	11.11%	25.17%	6.13%	3.16%	6.60%	5.60%	24.37%	31.69%
AR(2)TARCH(2,2)	5.99%	5.75%	12.04%	5.38%	47.31%	63.30%	5.56%	28.04%	52.20%	6.41%	0.81%	2.38%	5.76%	14.09%	24.02%
AR(3)TARCH(2,2)	5.88%	8.96%	21.94%	5.26%	61.47%	52.22%	5.61%	23.69%	40.96%	6.02%	5.14%	13.93%	5.76%	14.09%	24.02%
AR(4)TARCH(2,2)	5.94%	7.21%	18.20%	5.55%	30.02%	26.90%	5.56%	28.04%	44.80%	6.41%	0.81%	1.81%	5.76%	14.09%	24.02%
AR(0)EGARCH(0,1)	6.58%	0.28%	0.99%	5.89%	9.62%	14.77%	6.87%	0.05%	0.04%	6.35%	1.08%	2.99%	6.89%	0.04%	0.01%
AR(1)EGARCH(0,1)	6.64%	0.20%	0.72%	6.00%	6.14%	9.31%	7.03%	0.02%	0.02%	6.41%	0.81%	2.38%	7.11%	0.01%	0.00%
AR(2)EGARCH(0,1)	6.64%	0.20%	0.72%	5.94%	7.72%	11.80%	6.98%	0.02%	0.03%	6.52%	0.44%	1.46%	7.16%	0.01%	0.00%
AR(3)EGARCH(0,1)	6.75%	0.10%	0.42%	5.94%	7.72%	11.80%	7.08%	0.01%	0.01%	6.52%	0.44%	1.46%	7.22%	0.00%	0.00%
AR(4)EGARCH(0,1)	6.80%	0.07%	0.30%	5.94%	7.72%	11.80%	7.03%	0.02%	0.01%	6.52%	0.44%	1.46%	7.27%	0.00%	0.00%
AR(0)EGARCH(0,2)	6.42%	0.70%	2.45%	5.94%	7.72%	11.80%	6.38%	0.94%	1.37%	6.24%	1.87%	4.55%	6.79%	0.08%	0.03%
AR(1)EGARCH(0,2)	6.42%	0.70%	2.17%	6.00%	6.14%	9.31%	6.43%	0.70%	1.13%	6.30%	1.43%	3.71%	6.84%	0.06%	0.03%
AR(2)EGARCH(0,2)	6.48%	0.52%	1.70%	6.06%	4.85%	7.25%	6.54%	0.38%	0.74%	6.30%	1.43%	3.71%	6.84%	0.06%	0.03%
AR(3)EGARCH(0,2)	6.64%	0.20%	0.48%	6.06%	4.85%	7.25%	6.54%	0.38%	0.74%	6.46%	0.60%	1.45%	7.00%	0.02%	0.01%
AR(4)EGARCH(0,2)	6.64%	0.20%	0.48%	6.11%	3.79%	5.59%	6.65%	0.20%	0.46%	6.52%	0.44%	1.15%	7.05%	0.01%	0.01%
AR(0)EGARCH(1,1)	5.94%	7.21%	18.20%	5.26%	61.47%	52.22%	6.10%	3.58%	10.34%	6.63%	0.23%	0.20%	5.65%	20.47%	19.39%
AR(1)EGARCH(1,1)	5.99%	5.75%	14.91%	5.38%	47.31%	41.66%	6.16%	2.78%	8.34%	6.52%	0.44%	0.51%	5.76%	14.09%	16.62%
AR(2)EGARCH(1,1)	6.04%	4.55%	12.08%	5.66%	21.26%	18.55%	6.05%	4.56%	12.18%	6.74%	0.12%	0.31%	6.09%	3.77%	3.23%
AR(3)EGARCH(1,1)	6.04%	4.55%	12.08%	5.32%	54.14%	46.95%	6.10%	3.58%	10.34%	6.46%	0.60%	1.00%	5.92%	7.56%	7.74%
AR(4)EGARCH(1,1)	6.04%	4.55%	12.08%	5.38%	47.31%	41.66%	6.10%	3.58%	10.34%	6.46%	0.60%	1.45%	6.03%	4.80%	3.64%
AR(0)EGARCH(1,2)	5.99%	5.75%	15.32%	4.92%	88.26%	93.04%	6.05%	4.56%	12.18%	5.97%	6.48%	6.77%	5.76%	14.09	

Table (5.A). Normal Distribution, 2000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.94%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(1)GARCH(0,1)	3.99%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.45%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)GARCH(0,1)	4.10%	0.00%	0.00%	1.53%	3.85%	7.60%	3.49%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)GARCH(0,1)	4.10%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)GARCH(0,1)	4.05%	0.00%	0.00%	1.64%	1.32%	2.80%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(0)GARCH(0,2)	3.40%	0.00%	0.00%	1.64%	1.32%	2.80%	3.11%	0.00%	0.00%	2.90%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(1)GARCH(0,2)	3.29%	0.00%	0.00%	1.70%	0.73%	1.61%	3.11%	0.00%	0.00%	2.96%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(2)GARCH(0,2)	3.29%	0.00%	0.00%	1.58%	2.28%	4.70%	3.11%	0.00%	0.00%	2.96%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(3)GARCH(0,2)	3.29%	0.00%	0.00%	1.64%	1.32%	2.80%	3.16%	0.00%	0.00%	2.85%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(4)GARCH(0,2)	3.24%	0.00%	0.00%	1.64%	1.32%	2.80%	3.22%	0.00%	0.00%	2.85%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(0)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.70%	0.73%	1.61%	1.53%	3.56%	8.11%	1.86%	0.09%	0.15%	1.67%	0.82%	1.76%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.70%	0.73%	1.61%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(3)GARCH(1,1)	2.10%	0.00%	0.00%	1.70%	0.73%	1.61%	1.63%	1.23%	1.08%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(4)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(0)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(1)GARCH(1,2)	1.94%	0.03%	0.02%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(2)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.70%	0.64%	2.01%	1.67%	0.82%	1.76%
AR(3)GARCH(1,2)	2.21%	0.00%	0.00%	1.70%	0.73%	1.61%	1.69%	0.69%	2.14%	1.75%	0.35%	1.19%	1.67%	0.82%	1.76%
AR(4)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.67%	0.82%	1.76%
AR(0)GARCH(2,1)	2.21%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	0.71%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(1)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.80%	0.20%	0.28%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(2)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(3)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	5.40%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(4)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(0)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(1)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.81%	0.18%	0.26%	1.56%	2.46%	4.97%
AR(2)GARCH(2,2)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(3)GARCH(2,2)	2.05%	0.01%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(4)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.72%	0.45%	0.99%
AR(0)TARCH(0,1)	3.83%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.34%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(1)TARCH(0,1)	3.83%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)TARCH(0,1)	3.78%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.34%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)TARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.54%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(4)TARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.54%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(0)TARCH(0,2)	2.97%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(1)TARCH(0,2)	3.08%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(2)TARCH(0,2)	3.02%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,2)	3.13%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.79%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(4)TARCH(0,2)	3.13%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.79%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)TARCH(1,1)	1.94%	0.03%	0.14%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(1)TARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	1.75%	0.35%	0.43%	1.40%	10.22%	17.93%
AR(2)TARCH(1,1)	1.89%	0.06%	0.26%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	1.75%	0.35%	0.43%	1.40%	10.22%	17.93%
AR(3)TARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(4)TARCH(1,1)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(0)TARCH(1,2)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.75%	0.35%	0.43%	1.45%	6.56%	12.14%
AR(1)TARCH(1,2)	2.00%	0.01%	0.03%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.75%	0.35%	0.43%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	1.94%	0.03%	0.06%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.45%	6.56%	12.14%
AR(3)TARCH(1,2)	2.05%	0.01%	0.00%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(4)TARCH(1,2)	2.00%	0.01%	0.03%	1.58%	2.28%	4.70%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(0)TARCH(2,1)	1.78%	0.23%	0.53%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.92%	0.05%	0.20%	1.40%	10.22%	17.93%
AR(1)TARCH(2,1)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(2)TARCH(2,1)	1.78%	0.23%	0.84%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(3)TARCH(2,1)	1.78%	0.23%	0.53%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.45%	6.56%	12.14%
AR(4)TARCH(2,1)	2.00%	0.01%	0.03%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%	1.86%	0.09%	0.38%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(1)TARCH(2,2)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.85%	0.10%	0.41%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.74%	0.38%	1.28%	1.75%	0.35%	0.78%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.74%	0.38%	1.28%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(4)TARCH(2,2)	2.05%	0.01%	0.00%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(0)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.43%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(1)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(3)EGARCH(0,1)	4.10%	0.00%	0.00%	1.47%	6.27%	11.82%	3.60%	0.00%	0.00%	3.40%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)EGARCH(0,1)	4.05%	0.00%	0.00%	1.64%	1.32%	2.80%	3.60%	0.00%	0.00%	3.40%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(0)EGARCH(0,2)	3.99%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(1)EGARCH(0,2)	3.94%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.77%	0.00%	0.00%
AR(2)EGARCH(0,2)	3.99%	0.00%	0.00%	1.75%	0.40%	0.89%	3.43%	0.00%	0.00%	3.01%	0.00%	0.00%	3.77%	0.00%	0.00%
AR(3)EGARCH(0,2)	4.05%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(4)EGARCH(0,2)	4.05%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.07%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(0)EGARCH(1,1)	1.94%	0.03%	0.14%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.86%	0.09%	0.15%	1.45%	6.56%	12.14%
AR(1)EGARCH(1,1)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.80%	0.20%	0.28%	1.92%	0.05%	0.09%	1.51%	4.08%	7.91%
AR(2)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.69%	0.69%	0.71%	1.92%	0.05%	0.09%	1.56%	2.46%	4.97%
AR(3)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.86%	0.09%	0.15%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	1.78%	0.23%	0.84%	1.36%	15.12%	25.30%	1.69%	0.69%	0.71%	2.03%	0.01%	0.03%	1.40%	10.22%	17.93%
AR(1)EGARCH(1,2)	1.83%	0.12%	0.47%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.86%	0.09%	0.15%	1.40%	10.22%	17.93%
AR(2)EGARCH(1,2)	1.83%	0.12%	0.47%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.9					

Table (6.A). Normal Distribution, 1500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.78%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(1)GARCH(0,1)	3.78%	0.00%	0.00%	1.53%	3.85%	7.60%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(2)GARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)GARCH(0,1)	3.67%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)GARCH(0,1)	3.78%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(0)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.79%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(1)GARCH(0,2)	3.18%	0.00%	0.00%	1.58%	2.28%	4.70%	3.11%	0.00%	0.00%	2.79%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)GARCH(0,2)	3.24%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)GARCH(1,1)	2.16%	0.00%	0.00%	1.47%	6.27%	11.82%	1.69%	0.69%	2.14%	1.59%	2.00%	5.14%	1.56%	2.46%	4.97%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.70%	0.64%	2.01%	1.67%	0.82%	1.76%
AR(3)GARCH(1,1)	2.21%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(4)GARCH(1,1)	2.16%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(0)GARCH(1,2)	2.16%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	4.35%	1.59%	2.00%	5.14%	1.67%	0.82%	2.48%
AR(1)GARCH(1,2)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.72%	0.45%	1.49%
AR(2)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.62%	1.44%	3.96%
AR(3)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(4)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.56%	2.46%	4.97%
AR(0)GARCH(2,1)	2.21%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	5.40%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(1)GARCH(2,1)	2.10%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	5.40%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(2)GARCH(2,1)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.58%	2.12%	5.40%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(3)GARCH(2,1)	2.32%	0.00%	0.00%	1.41%	9.90%	17.66%	1.58%	2.12%	5.40%	1.64%	1.15%	3.28%	1.72%	0.45%	0.99%
AR(4)GARCH(2,1)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.58%	2.12%	5.40%	1.75%	0.35%	1.19%	1.72%	0.45%	0.99%
AR(0)GARCH(2,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.70%	0.64%	2.01%	1.62%	1.44%	3.96%
AR(1)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%	1.64%	1.15%	3.28%	1.45%	6.56%	12.14%
AR(2)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(3)GARCH(2,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(4)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(0)TARCH(0,1)	3.62%	0.00%	0.00%	1.53%	3.85%	7.60%	3.32%	0.00%	0.00%	3.29%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(1)TARCH(0,1)	3.56%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(2)TARCH(0,1)	3.56%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(3)TARCH(0,1)	3.56%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.23%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(4)TARCH(0,1)	3.56%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(0)TARCH(0,2)	3.13%	0.00%	0.00%	1.53%	3.85%	7.60%	2.94%	0.00%	0.00%	2.96%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(1)TARCH(0,2)	3.13%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.96%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)TARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.90%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.79%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)TARCH(0,2)	3.18%	0.00%	0.00%	1.58%	2.28%	4.70%	3.00%	0.00%	0.00%	2.85%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(0)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.74%	0.38%	0.84%	1.70%	0.64%	1.40%	1.45%	6.56%	12.14%
AR(1)TARCH(1,1)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.74%	0.38%	0.84%	1.81%	0.18%	0.42%	1.40%	10.22%	17.93%
AR(2)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.40%	10.22%	17.93%
AR(3)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(4)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(0)TARCH(1,2)	2.05%	0.01%	0.02%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.70%	0.64%	2.01%	1.45%	6.56%	12.14%
AR(1)TARCH(1,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.70%	0.64%	2.01%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	1.50%	1.70%	0.64%	2.01%	1.56%	2.46%	6.10%
AR(3)TARCH(1,2)	2.16%	0.00%	0.00%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(4)TARCH(1,2)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.97%	0.02%	0.05%	1.51%	4.08%	7.91%
AR(0)TARCH(2,1)	2.10%	0.00%	0.02%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.68%	1.45%	6.56%	12.14%
AR(1)TARCH(2,1)	2.05%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(2)TARCH(2,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(3)TARCH(2,1)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(4)TARCH(2,1)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(1)TARCH(2,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	1.50%	1.75%	0.35%	0.43%	1.45%	6.56%	12.14%
AR(2)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	2.05%	0.01%	0.02%	1.36%	15.12%	25.30%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(4)TARCH(2,2)	2.00%	0.01%	0.03%	1.41%	9.90%	17.66%	1.63%	1.23%	3.46%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(0)EGARCH(0,1)	3.62%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(1)EGARCH(0,1)	3.72%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.23%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(2)EGARCH(0,1)	3.72%	0.00%	0.00%	1.53%	3.85%	8.47%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(3)EGARCH(0,1)	3.78%	0.00%	0.00%	1.47%	6.27%	11.82%	3.60%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(4)EGARCH(0,1)	3.78%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(0)EGARCH(0,2)	4.05%	0.00%	0.00%	1.81%	0.21%	0.48%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(1)EGARCH(0,2)	4.10%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(2)EGARCH(0,2)	4.05%	0.00%	0.00%	1.87%	0.11%	0.25%	3.43%	0.00%	0.00%	3.01%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(3)EGARCH(0,2)	4.10%	0.00%	0.00%	1.87%	0.11%	0.25%	3.38%	0.00%	0.00%	2.96%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.99%	0.00%	0.00%	1.87%	0.11%	0.25%	3.38%	0.00%	0.00%	2.96%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(0)EGARCH(1,1)	2.16%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(1)EGARCH(1,1)	2.21%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.86%	0.09%	0.15%	1.56%	2.46%	4.97%
AR(2)EGARCH(1,1)	2.16%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.97%	0.02%	0.05%	1.51%	4.08%	7.91%
AR(3)EGARCH(1,1)	2.05%	0.01%	0.02%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.92%	0.05%	0.09%	1.56%	2.46%	4.97%
AR(4)EGARCH(1,1)	2.16%	0.00%	0.00%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.86%	0.09%	0.15%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	1.89%	0.06%	0.11%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.81%	0.18%	0.68%	1.62%	1.44%	3.01%
AR(1)EGARCH(1,2)	2.00%	0.01%	0.03%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.62%	1.44%	3.01%
AR(2)EGARCH(1,2)	1.89%	0.06%	0.11%	1.30%	22.33%	34.71%	1.74%	0.38%</							

Table (7.A). Normal Distribution, 1000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.18%	0.00%	0.00%	1.58%	2.28%	5.65%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(1)GARCH(0,1)	3.13%	0.00%	0.00%	1.58%	2.28%	5.65%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	2.96%	0.00%	0.00%
AR(2)GARCH(0,1)	3.18%	0.00%	0.00%	1.53%	3.85%	8.47%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(3)GARCH(0,1)	3.18%	0.00%	0.00%	1.53%	3.85%	8.47%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(4)GARCH(0,1)	3.13%	0.00%	0.00%	1.53%	3.85%	8.47%	3.27%	0.00%	0.00%	3.29%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(0)GARCH(0,2)	3.08%	0.00%	0.00%	1.47%	6.27%	11.82%	2.89%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(1)GARCH(0,2)	3.08%	0.00%	0.00%	1.58%	2.28%	4.70%	2.89%	0.00%	0.00%	2.63%	0.00%	0.00%	2.53%	0.00%	0.00%
AR(2)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	2.94%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(3)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	2.83%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(4)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	3.00%	0.00%	0.00%	2.63%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(0)GARCH(1,1)	2.05%	0.01%	0.00%	1.64%	1.32%	3.62%	1.80%	0.20%	0.73%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(3)GARCH(1,1)	2.16%	0.00%	0.00%	1.70%	0.73%	2.23%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(1,1)	2.05%	0.01%	0.02%	1.70%	0.73%	2.23%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.62%	1.44%	3.01%
AR(0)GARCH(1,2)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(1)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.59%	2.00%	4.11%	1.45%	6.56%	12.14%
AR(2)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(3)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	1.28%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(4)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(0)GARCH(2,1)	2.05%	0.01%	0.00%	1.58%	2.28%	4.70%	1.69%	0.69%	2.14%	1.81%	0.18%	0.68%	1.51%	4.08%	7.91%
AR(1)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	3.46%	1.75%	0.35%	0.78%	1.56%	2.46%	4.97%
AR(2)GARCH(2,1)	2.05%	0.01%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	3.46%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(3)GARCH(2,1)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	3.46%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(0)GARCH(2,2)	2.21%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.51%	4.08%	7.91%
AR(1)GARCH(2,2)	2.21%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.45%	6.56%	12.14%
AR(2)GARCH(2,2)	2.21%	0.00%	0.00%	1.53%	3.85%	8.47%	1.63%	1.23%	2.60%	1.59%	2.00%	4.11%	1.51%	4.08%	7.91%
AR(3)GARCH(2,2)	2.27%	0.00%	0.00%	1.47%	6.27%	12.19%	1.69%	0.69%	1.50%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(2,2)	2.27%	0.00%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	2.60%	1.75%	0.35%	0.78%	1.51%	4.08%	7.91%
AR(0)TARCH(0,1)	3.18%	0.00%	0.00%	1.58%	2.28%	5.65%	3.05%	0.00%	0.00%	3.29%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(1)TARCH(0,1)	3.08%	0.00%	0.00%	1.58%	2.28%	4.70%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(2)TARCH(0,1)	3.08%	0.00%	0.00%	1.53%	3.85%	7.60%	3.16%	0.00%	0.00%	3.07%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,1)	3.02%	0.00%	0.00%	1.53%	3.85%	7.60%	3.16%	0.00%	0.00%	3.12%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)TARCH(0,1)	3.02%	0.00%	0.00%	1.64%	1.32%	3.62%	3.16%	0.00%	0.00%	3.01%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)TARCH(0,2)	2.91%	0.00%	0.00%	1.47%	6.27%	11.82%	2.62%	0.00%	0.00%	2.79%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(1)TARCH(0,2)	2.97%	0.00%	0.00%	1.41%	9.90%	17.66%	2.62%	0.00%	0.00%	2.74%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(2)TARCH(0,2)	2.86%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.74%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(3)TARCH(0,2)	3.13%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(4)TARCH(0,2)	3.13%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.68%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(0)TARCH(1,1)	2.05%	0.01%	0.03%	1.58%	2.28%	4.70%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.40%	10.22%	17.93%
AR(1)TARCH(1,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.45%	6.56%	12.14%
AR(2)TARCH(1,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(3)TARCH(1,1)	1.94%	0.03%	0.14%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.51%	4.08%	7.91%
AR(4)TARCH(1,1)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(0)TARCH(1,2)	1.83%	0.12%	0.19%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.40%	10.22%	17.93%
AR(1)TARCH(1,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.56%	2.46%	4.97%
AR(3)TARCH(1,2)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(4)TARCH(1,2)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.56%	2.46%	4.97%
AR(0)TARCH(2,1)	2.00%	0.01%	0.07%	1.58%	2.28%	4.70%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.35%	15.43%	25.43%
AR(1)TARCH(2,1)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.92%	0.05%	0.11%	1.45%	6.56%	12.14%
AR(2)TARCH(2,1)	2.00%	0.01%	0.07%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.56%	2.46%	4.97%
AR(3)TARCH(2,1)	1.94%	0.03%	0.14%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	2.03%	0.01%	0.05%	1.56%	2.46%	4.97%
AR(4)TARCH(2,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.86%	0.09%	0.22%	1.62%	1.44%	3.01%
AR(0)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(1)TARCH(2,2)	2.00%	0.01%	0.03%	1.41%	9.90%	17.66%	1.58%	2.12%	4.35%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(3)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.92%	0.05%	0.20%	1.56%	2.46%	4.97%
AR(4)TARCH(2,2)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(0)EGARCH(0,1)	3.02%	0.00%	0.00%	1.53%	3.85%	8.47%	3.00%	0.00%	0.00%	3.01%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(1)EGARCH(0,1)	3.08%	0.00%	0.00%	1.47%	6.27%	12.19%	3.00%	0.00%	0.00%	3.07%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)EGARCH(0,1)	3.08%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(3)EGARCH(0,1)	3.02%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)EGARCH(0,1)	3.02%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)EGARCH(0,2)	3.62%	0.00%	0.00%	1.64%	1.32%	2.80%	3.05%	0.00%	0.00%	2.85%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(1)EGARCH(0,2)	3.62%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.74%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(2)EGARCH(0,2)	3.67%	0.00%	0.00%	1.64%	1.32%	2.80%	3.00%	0.00%	0.00%	2.74%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(3)EGARCH(0,2)	3.62%	0.00%	0.00%	1.58%	2.28%	4.70%	3.00%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.72%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.85%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(0)EGARCH(1,1)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	2.14%	1.86%	0.09%	0.15%	1.29%	22.56%	34.59%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.14%	1.30%	22.33%	34.71%	1.69%	0.69%	2.14%	1.86%	0.09%	0.15%	1.40%	10.22%	17.93%
AR(2)EGARCH(1,1)	1.89%	0.06%	0.26%	1.30%	22.33%	34.71%	1.69%	0.69%	2.14%	1.81%	0.18%	0.26%	1.35%	15.43%	25.43%
AR(3)EGARCH(1,1)	1.89%	0.06%	0.26%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.92%	0.05%	0.09%	1.35%	15.43%	25.43%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.14%	1.36%	15.12%	25.30%	1.80%	0.20%	0.73%	1.92%	0.05%	0.02%	1.40%	10.22%	17.93%
AR(0)EGARCH(1,2)	1.62%	1.40%	1.19%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.53%	3.37%	6.67%	1.40%	10.22%	17.93%
AR(1)EGARCH(1,2)	1.62%	1.40%	1.19%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%	1.48%	5.50%	10.42%	1.29%	22.56%	34.59%
AR(2)EGARCH(1,2)	1.62%	1.40%	1.19%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%						

Table (8.A). Normal Distribution, 500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	2.27%	0.00%	0.00%	1.58%	2.28%	5.65%	2.67%	0.00%	0.00%	2.52%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	1.64%	1.32%	3.62%	2.67%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(2)GARCH(0,1)	2.43%	0.00%	0.00%	1.70%	0.73%	0.72%	2.72%	0.00%	0.00%	2.57%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(3)GARCH(0,1)	2.43%	0.00%	0.00%	1.75%	0.40%	0.46%	2.72%	0.00%	0.00%	2.46%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(4)GARCH(0,1)	2.48%	0.00%	0.00%	1.70%	0.73%	2.23%	2.72%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(0)GARCH(0,2)	2.37%	0.00%	0.00%	1.53%	3.85%	8.47%	2.34%	0.00%	0.00%	2.52%	0.00%	0.00%	2.26%	0.00%	0.00%
AR(1)GARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	8.47%	2.34%	0.00%	0.00%	2.35%	0.00%	0.00%	2.37%	0.00%	0.00%
AR(2)GARCH(0,2)	2.37%	0.00%	0.00%	1.58%	2.28%	5.65%	2.45%	0.00%	0.00%	2.46%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(3)GARCH(0,2)	2.37%	0.00%	0.00%	1.64%	1.32%	3.62%	2.45%	0.00%	0.00%	2.52%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(4)GARCH(0,2)	2.37%	0.00%	0.00%	1.64%	1.32%	3.62%	2.45%	0.00%	0.00%	2.52%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(0)GARCH(1,1)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	1.64%	1.15%	2.45%	1.62%	1.44%	3.01%
AR(2)GARCH(1,1)	1.94%	0.03%	0.06%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(3)GARCH(1,1)	2.05%	0.01%	0.02%	1.58%	2.28%	4.70%	1.36%	13.92%	23.40%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(4)GARCH(1,1)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.36%	13.92%	23.40%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(0)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.74%	0.38%	0.84%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(1)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.69%	0.69%	1.50%	1.53%	3.37%	7.75%	1.56%	2.46%	4.97%
AR(2)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.53%	3.56%	7.01%	1.42%	8.70%	15.60%	1.88%	0.06%	0.15%
AR(3)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.58%	2.12%	4.35%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(4)GARCH(1,2)	2.10%	0.00%	0.01%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(0)GARCH(2,1)	2.05%	0.01%	0.02%	1.64%	1.32%	3.62%	1.74%	0.38%	0.84%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(1)GARCH(2,1)	2.05%	0.01%	0.02%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.64%	1.15%	3.28%	1.51%	4.08%	7.91%
AR(2)GARCH(2,1)	2.05%	0.01%	0.02%	1.64%	1.32%	3.62%	1.53%	3.56%	7.01%	1.81%	0.18%	0.68%	1.62%	1.44%	3.01%
AR(3)GARCH(2,1)	2.05%	0.01%	0.02%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(4)GARCH(2,1)	2.00%	0.01%	0.03%	1.41%	9.90%	16.79%	1.47%	5.79%	10.90%	1.70%	0.64%	2.01%	1.62%	1.44%	3.01%
AR(0)GARCH(2,2)	2.21%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	0.84%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(1)GARCH(2,2)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	1.50%	1.70%	0.64%	2.01%	1.62%	1.44%	3.01%
AR(2)GARCH(2,2)	2.10%	0.00%	0.01%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.75%	0.35%	1.19%	1.72%	0.45%	0.99%
AR(3)GARCH(2,2)	2.16%	0.00%	0.00%	1.70%	0.73%	2.23%	1.47%	5.79%	10.90%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(4)GARCH(2,2)	2.21%	0.00%	0.00%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(0)TARCH(0,1)	2.32%	0.00%	0.00%	1.64%	1.32%	2.80%	2.78%	0.00%	0.00%	2.41%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(1)TARCH(0,1)	2.27%	0.00%	0.00%	1.70%	0.73%	1.61%	2.78%	0.00%	0.00%	2.46%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(2)TARCH(0,1)	2.32%	0.00%	0.00%	1.70%	0.73%	1.61%	2.62%	0.00%	0.00%	2.57%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(3)TARCH(0,1)	2.43%	0.00%	0.00%	1.70%	0.73%	1.61%	2.67%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(4)TARCH(0,1)	2.48%	0.00%	0.00%	1.70%	0.73%	1.61%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(0)TARCH(0,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.14%	0.00%	0.01%	2.37%	0.00%	0.00%
AR(1)TARCH(0,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.19%	0.00%	0.00%	2.53%	0.00%	0.00%
AR(2)TARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.19%	0.00%	0.00%	2.48%	0.00%	0.00%
AR(3)TARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.08%	0.01%	0.03%	2.48%	0.00%	0.00%
AR(4)TARCH(0,2)	2.54%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.14%	0.00%	0.01%	2.48%	0.00%	0.00%
AR(0)TARCH(1,1)	1.83%	0.12%	0.28%	1.53%	3.85%	7.60%	1.58%	2.12%	4.35%	2.25%	0.00%	0.00%	1.35%	15.43%	25.43%
AR(1)TARCH(1,1)	1.83%	0.12%	0.28%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	2.14%	0.00%	0.01%	1.35%	15.43%	25.43%
AR(2)TARCH(1,1)	1.89%	0.06%	0.14%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	2.08%	0.01%	0.03%	1.51%	4.08%	7.91%
AR(3)TARCH(1,1)	1.89%	0.06%	0.14%	1.64%	1.32%	2.80%	1.47%	5.79%	10.90%	2.19%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(4)TARCH(1,1)	1.94%	0.03%	0.07%	1.53%	3.85%	7.60%	1.47%	5.79%	10.90%	2.25%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)TARCH(1,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	2.03%	0.01%	0.03%	1.56%	2.46%	4.97%
AR(1)TARCH(1,2)	2.05%	0.01%	0.02%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.58%	2.12%	4.35%	2.03%	0.01%	0.03%	1.51%	4.08%	7.91%
AR(3)TARCH(1,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.45%	6.56%	12.14%
AR(4)TARCH(1,2)	1.89%	0.06%	0.11%	1.53%	3.85%	7.60%	1.53%	3.56%	7.01%	2.35%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)TARCH(2,1)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(1)TARCH(2,1)	1.94%	0.03%	0.07%	1.58%	2.28%	4.70%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(2,1)	1.94%	0.03%	0.07%	1.41%	9.90%	17.66%	1.47%	5.79%	10.90%	2.25%	0.00%	0.00%	1.56%	2.46%	4.97%
AR(3)TARCH(2,1)	1.94%	0.03%	0.07%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	2.30%	0.00%	0.00%	1.51%	4.08%	7.91%
AR(4)TARCH(2,1)	1.89%	0.06%	0.14%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	2.19%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.19%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(1)TARCH(2,2)	1.89%	0.06%	0.11%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.47%	5.79%	10.90%	2.14%	0.00%	0.01%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.25%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(4)TARCH(2,2)	2.00%	0.01%	0.03%	1.75%	0.40%	0.89%	1.53%	3.56%	7.01%	2.30%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)EGARCH(0,1)	2.10%	0.00%	0.02%	1.70%	0.73%	2.23%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)EGARCH(0,1)	2.10%	0.00%	0.02%	1.75%	0.40%	1.32%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.85%	0.00%	0.00%
AR(2)EGARCH(0,1)	2.05%	0.01%	0.03%	1.81%	0.21%	0.76%	2.62%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(3)EGARCH(0,1)	2.10%	0.00%	0.01%	1.81%	0.21%	0.76%	2.67%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)EGARCH(0,1)	2.10%	0.00%	0.01%	1.81%	0.21%	0.76%	2.62%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)EGARCH(0,2)	2.70%	0.00%	0.00%	1.75%	0.40%	1.32%	2.51%	0.00%	0.00%	2.41%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.70%	0.00%	0.00%	1.75%	0.40%	1.32%	2.56%	0.00%	0.00%	2.35%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(2)EGARCH(0,2)	2.86%	0.00%	0.00%	1.75%	0.40%	1.32%	2.78%	0.00%	0.00%	2.41%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(3)EGARCH(0,2)	2.91%	0.00%	0.00%	1.70%	0.73%	2.23%	2.78%	0.00%	0.00%	2.30%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.02%	0.00%	0.00%	1.75%	0.40%	1.32%	2.67%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)EGARCH(1,1)	1.94%	0.03%	0.14%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	1.97%	0.02%	0.05%	1.56%	2.46%	4.97%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.07%	1.64%	1.32%	2.80%	1.63%	1.23%	2.60%	1.86%	0.09%	0.38%	1.56%	2.46%	4.97%
AR(2)EGARCH(1,1)	1.83%	0.12%	0.28%	1.58%	2.28%	4.70%	1.58%	2.12%	4.35%	2.03%	0.01%	0.03%	1.56%	2.46%	4.97%
AR(3)EGARCH(1,1)	1.89%	0.06%	0.14%	1.64%	1.32%	2.80%	1.63%	1.23%	2.60%	2.08%	0.01%	0.01%	1.51%	4.08%	7.91%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.07%	1.70%	0.73%	1.61%	1.58%	2.12%	4.35%	2.08%	0.01%	0.01%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	2.21%	0.00%	0.00%	1.81%	0.21%	0.48%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(1)EGARCH(1,2)	2.05%	0.01%	0.02%	1.98%	0.03%	0.12%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(2)EGARCH(1,2)	2.16%	0.00%	0.00%	1.87%	0.11%	0.25%	1.63%	1.23%	2.60%	1.86%	0.09%				

Table (9.1.A). Student-t Distribution, 2000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	5.07%	88.58%	93.39%	2.49%	0.00%	0.00%	5.45%	38.34%	63.29%	5.48%	35.72%	20.97%	5.76%	14.09%	5.41%
AR(1)GARCH(0,2)	5.02%	97.03%	65.26%	2.94%	0.00%	0.00%	4.80%	68.60%	81.39%	5.42%	41.44%	35.91%	5.87%	9.38%	4.80%
AR(1)GARCH(1,1)	3.35%	0.05%	0.20%	3.51%	0.24%	0.97%	4.20%	10.46%	23.39%	4.71%	56.57%	72.07%	4.85%	76.04%	86.48%
AR(1)GARCH(1,2)	3.35%	0.05%	0.20%	3.51%	0.24%	0.97%	4.14%	8.25%	12.76%	4.71%	56.57%	51.27%	4.90%	84.34%	12.31%
AR(1)GARCH(2,1)	3.35%	0.05%	0.20%	3.57%	0.36%	1.37%	-	-	-	4.71%	56.57%	72.07%	4.85%	76.04%	86.48%
AR(1)GARCH(2,2)	3.51%	0.19%	0.69%	3.51%	0.24%	0.97%	4.09%	6.43%	9.85%	4.65%	49.39%	65.60%	4.79%	67.99%	61.72%
AR(1)TARCH(0,1)	4.70%	54.31%	38.90%	2.38%	0.00%	0.00%	5.40%	44.30%	69.60%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	3.23%	0.03%	0.02%	5.12%	81.03%	84.97%	-	-	-	-	-	-
AR(1)TARCH(1,1)	3.62%	0.41%	1.46%	3.45%	0.16%	0.08%	4.31%	16.23%	34.13%	4.82%	72.15%	83.22%	4.74%	60.25%	82.85%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	3.62%	0.41%	1.46%	3.45%	0.16%	0.08%	4.31%	16.23%	34.13%	4.93%	88.87%	90.64%	4.90%	84.34%	71.39%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	4.91%	86.00%	91.00%	2.32%	0.00%	0.00%	5.40%	44.30%	67.49%	5.48%	35.72%	11.07%	6.19%	2.28%	0.02%
AR(1)EGARCH(0,2)	5.23%	64.53%	22.75%	3.06%	0.01%	0.01%	4.96%	93.59%	5.58%	5.37%	47.69%	36.59%	6.19%	2.28%	0.33%
AR(1)EGARCH(1,1)	4.26%	13.59%	30.78%	3.68%	0.76%	0.23%	4.36%	19.86%	40.25%	5.15%	77.29%	56.51%	5.82%	11.54%	26.10%
AR(1)EGARCH(1,2)	4.43%	24.73%	47.98%	3.51%	0.24%	0.10%	4.69%	53.38%	78.53%	5.15%	77.29%	56.51%	5.65%	20.47%	29.27%
AR(1)EGARCH(2,1)	4.43%	24.73%	47.98%	3.74%	1.08%	0.29%	4.69%	53.38%	78.53%	5.31%	54.44%	56.38%	5.39%	45.19%	55.60%

Table (9.2.A). Student-t Distribution, 1500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	5.40%	43.90%	67.63%	2.89%	0.00%	0.01%	5.56%	28.04%	32.60%	5.31%	54.44%	36.78%	5.60%	24.37%	20.54%
AR(1)GARCH(0,2)	5.02%	97.03%	89.77%	3.06%	0.01%	0.01%	5.23%	65.13%	77.44%	5.42%	41.44%	21.17%	5.71%	17.05%	10.72%
AR(1)GARCH(1,1)	3.56%	0.28%	1.02%	3.34%	0.07%	0.30%	4.47%	28.79%	50.62%	4.71%	56.57%	51.27%	5.12%	81.96%	58.05%
AR(1)GARCH(1,2)	3.62%	0.41%	1.46%	3.51%	0.24%	0.97%	4.41%	24.04%	46.74%	4.82%	72.15%	62.33%	4.95%	92.78%	28.59%
AR(1)GARCH(2,1)	3.51%	0.19%	0.69%	3.34%	0.07%	0.30%	-	-	-	4.71%	56.57%	51.27%	5.06%	90.27%	56.17%
AR(1)GARCH(2,2)	3.83%	1.63%	5.29%	3.45%	0.16%	0.66%	4.47%	28.79%	53.44%	4.87%	80.42%	67.20%	4.95%	92.78%	50.39%
AR(1)TARCH(0,1)	-	-	-	2.94%	0.00%	0.00%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	3.17%	0.02%	0.01%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.57%	0.36%	0.14%	4.25%	13.10%	22.04%	4.65%	49.39%	45.46%	5.06%	90.27%	93.61%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	3.57%	0.36%	0.14%	4.20%	10.46%	19.14%	4.65%	49.39%	65.60%	5.06%	90.27%	79.77%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	5.45%	38.00%	49.95%	2.77%	0.00%	0.00%	5.94%	7.24%	4.13%	5.31%	54.44%	20.68%	6.09%	3.77%	0.39%
AR(1)EGARCH(0,2)	5.45%	38.00%	49.95%	3.06%	0.01%	0.01%	5.45%	38.34%	5.24%	5.48%	35.72%	49.70%	5.60%	24.37%	2.62%
AR(1)EGARCH(1,1)	5.07%	88.58%	62.04%	3.51%	0.24%	0.10%	5.18%	72.92%	80.67%	5.04%	94.02%	52.72%	5.76%	14.09%	29.99%
AR(1)EGARCH(1,2)	4.86%	77.66%	70.10%	3.62%	0.53%	1.19%	5.07%	89.37%	93.21%	5.09%	85.56%	7.40%	5.65%	20.47%	38.15%
AR(1)EGARCH(2,1)	4.96%	94.47%	67.74%	3.45%	0.16%	0.08%	5.18%	72.92%	89.24%	4.98%	97.43%	49.83%	5.87%	9.38%	13.53%

Table (9.3.A). Student-t Distribution, 1000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	4.86%	77.66%	87.14%	3.28%	0.04%	0.15%	5.29%	57.73%	37.26%	5.31%	54.44%	36.78%	5.55%	28.78%	2.56%
AR(1)GARCH(0,2)	4.75%	61.73%	83.70%	3.34%	0.07%	0.22%	5.07%	89.37%	78.58%	5.37%	47.69%	21.07%	5.76%	14.09%	2.60%
AR(1)GARCH(1,1)	3.78%	1.18%	3.94%	3.28%	0.04%	0.19%	4.58%	40.00%	66.74%	4.93%	88.87%	90.64%	5.22%	66.08%	59.45%
AR(1)GARCH(1,2)	3.78%	1.18%	2.91%	3.51%	0.24%	0.97%	4.52%	34.11%	49.37%	5.04%	94.02%	30.47%	5.22%	66.08%	21.01%
AR(1)GARCH(2,1)	3.83%	1.63%	5.29%	3.40%	0.10%	0.45%	-	-	-	-	-	-	5.17%	73.87%	59.15%
AR(1)GARCH(2,2)	3.89%	2.22%	6.98%	3.57%	0.36%	1.37%	4.58%	40.00%	56.23%	-	-	-	5.33%	51.69%	38.65%
AR(1)TARCH(0,1)	-	-	-	3.23%	0.03%	0.10%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	3.34%	0.07%	0.04%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.28%	0.04%	0.15%	4.36%	19.86%	40.25%	-	-	-	5.28%	58.67%	58.93%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	3.40%	0.10%	0.32%	-	-	-	-	-	-	5.33%	51.69%	57.63%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	5.07%	88.58%	93.39%	3.11%	0.01%	0.04%	5.23%	65.13%	9.57%	5.31%	54.44%	56.38%	5.76%	14.09%	2.60%
AR(1)EGARCH(0,2)	5.13%	80.30%	83.88%	3.40%	0.10%	0.32%	5.56%	28.04%	5.47%	5.37%	47.69%	21.07%	5.71%	17.05%	1.12%
AR(1)EGARCH(1,1)	4.70%	54.31%	66.57%	3.79%	1.51%	0.36%	5.29%	57.73%	69.86%	-	-	-	5.76%	14.09%	31.79%
AR(1)EGARCH(1,2)	4.64%	47.33%	63.62%	3.79%	1.51%	2.63%	4.96%	93.59%	92.12%	5.09%	85.56%	32.57%	5.76%	14.09%	10.07%
AR(1)EGARCH(2,1)	4.75%	61.73%	68.67%	3.85%	2.08%	3.32%	5.40%	44.30%	69.60%	5.04%	94.02%	93.25%	5.60%	24.37%	20.54%

Table (9.4.A). Student-t Distribution, 500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	4.16%	8.61%	7.46%	3.34%	0.07%	0.30%	4.63%	46.44%	63.08%	-	-	-	-	-	-
AR(1)GARCH(0,2)	4.05%	5.20%	8.10%	3.74%	1.08%	3.55%	5.18%	72.92%	57.66%	-	-	-	-	-	-
AR(1)GARCH(1,1)	3.83%	1.63%	4.03%	3.74%	1.08%	3.52%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(1,2)	3.89%	2.22%	3.25%	-	-	-	4.47%	28.79%	42.68%	-	-	-	-	-	-
AR(1)GARCH(2,1)	3.89%	2.22%	5.49%	3.85%	2.08%	6.47%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	3.94%	2.99%	4.48%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.57%	0.36%	0.88%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	4.26%	13.59%	21.56%	3.28%	0.04%	0.15%	5.07%	89.37%	32.24%	-	-	-	-	-	-
AR(1)EGARCH(0,2)	4.91%	86.00%	90.54%	3.79%	1.51%	4.70%	5.01%	97.86%	30.00%	-	-	-	-	-	-
AR(1)EGARCH(1,1)	4.75%	61.73%	77.33%	3.62%	0.53%	1.92%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(1,2)	5.02%	97.03%	78.16%	4.07%	6.56%	14.69%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(2,1)	4.75%	61.73%	77.33%	3.62%	0.53%	1.92%	-	-	-	-	-	-	-	-	-

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (10.1.A). Student-t Distribution, 2000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.70%	17.25%	35.72%	0.23%	0.01%	0.04%	1.04%	87.95%	80.19%	1.53%	3.37%	7.75%	1.88%	0.06%	0.15%
AR(1)GARCH(0,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	0.98%	93.43%	82.56%	1.48%	5.50%	11.22%	1.45%	6.56%	12.14%
AR(1)GARCH(1,1)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	0.60%	6.24%	16.40%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(1,2)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	0.54%	3.20%	9.45%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,1)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	-	-	-	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,2)	0.65%	10.32%	24.35%	0.23%	0.01%	0.04%	0.54%	3.20%	9.45%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)TARCH(0,1)	0.70%	17.25%	35.72%	0.23%	0.01%	0.04%	0.93%	74.84%	32.98%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.28%	0.03%	0.16%	0.93%	74.84%	80.27%	-	-	-	-	-	-
AR(1)TARCH(1,1)	0.59%	5.72%	15.25%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.04%	86.27%	79.83%	0.75%	26.52%	47.96%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	0.59%	5.72%	15.25%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.04%	86.27%	79.83%	0.75%	26.52%	47.96%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.65%	10.32%	24.35%	0.23%	0.01%	0.04%	1.09%	70.28%	42.48%	1.53%	3.37%	7.75%	1.72%	0.45%	0.99%
AR(1)EGARCH(0,2)	0.86%	54.53%	71.82%	0.40%	0.37%	1.42%	1.36%	13.92%	23.40%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(1,1)	0.76%	26.90%	48.43%	0.11%	0.00%	0.00%	0.71%	18.54%	37.65%	1.26%	28.39%	32.06%	1.24%	31.92%	45.06%
AR(1)EGARCH(1,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.26%	28.39%	32.06%	1.18%	43.70%	56.11%
AR(1)EGARCH(2,1)	0.81%	39.40%	61.02%	0.11%	0.00%	0.00%	0.60%	6.24%	16.40%	1.20%	39.41%	37.05%	1.18%	43.70%	56.11%

Table (10.2.A). Student-t Distribution, 1500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	1.31%	20.56%	27.08%	1.48%	5.50%	11.22%	1.78%	0.24%	0.54%
AR(1)GARCH(0,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	1.25%	29.39%	42.50%	1.31%	19.78%	26.41%	1.45%	6.56%	12.14%
AR(1)GARCH(1,1)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(1,2)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,1)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	-	-	-	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,2)	0.70%	17.25%	35.72%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)TARCH(0,1)	-	-	-	0.28%	0.03%	0.16%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.99%	95.11%	38.42%	0.81%	38.90%	60.57%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.23%	0.01%	0.04%	0.82%	41.70%	63.05%	1.15%	52.91%	40.67%	0.81%	38.90%	60.57%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.76%	26.90%	48.43%	0.28%	0.03%	0.16%	1.25%	29.39%	32.70%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(0,2)	0.97%	90.11%	82.35%	0.51%	2.21%	6.92%	1.47%	5.79%	10.90%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(1,1)	1.08%	73.47%	75.08%	0.23%	0.01%	0.04%	1.09%	70.28%	73.77%	1.53%	3.37%	2.09%	1.45%	6.56%	12.14%
AR(1)EGARCH(1,2)	0.92%	71.72%	79.27%	0.17%	0.00%	0.01%	1.09%	70.28%	73.77%	1.53%	3.37%	2.09%	1.40%	10.22%	17.56%
AR(1)EGARCH(2,1)	1.08%	73.47%	42.81%	0.11%	0.00%	0.00%	1.20%	40.63%	53.58%	1.53%	3.37%	2.09%	1.40%	10.22%	17.93%

Table (10.3.A). Student-t Distribution, 1000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.97%	90.11%	82.35%	0.45%	0.96%	3.34%	1.31%	20.56%	27.08%	1.64%	1.15%	1.02%	1.62%	1.44%	3.01%
AR(1)GARCH(0,2)	0.97%	90.11%	82.35%	0.34%	0.12%	0.52%	1.42%	9.12%	16.29%	1.37%	13.33%	20.76%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	0.92%	71.72%	79.27%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.88%	58.71%	74.25%	1.18%	43.70%	56.11%
AR(1)GARCH(1,2)	0.86%	54.53%	71.82%	0.17%	0.00%	0.01%	0.93%	74.84%	80.27%	0.99%	95.11%	82.61%	1.13%	57.88%	66.64%
AR(1)GARCH(2,1)	0.86%	54.53%	71.82%	0.17%	0.00%	0.01%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)GARCH(2,2)	0.97%	90.11%	82.35%	0.17%	0.00%	0.01%	0.93%	74.84%	80.27%	-	-	-	1.13%	57.88%	66.64%
AR(1)TARCH(0,1)	-	-	-	0.45%	0.96%	3.34%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.34%	0.12%	0.52%	0.82%	41.70%	63.05%	-	-	-	1.02%	92.04%	80.91%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	1.02%	92.04%	80.91%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.86%	54.53%	71.82%	0.51%	2.21%	6.92%	1.42%	9.12%	16.15%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(1)EGARCH(0,2)	1.13%	57.23%	66.25%	0.51%	2.21%	6.92%	1.63%	1.23%	3.46%	1.70%	0.64%	0.67%	1.56%	2.46%	4.97%
AR(1)EGARCH(1,1)	0.97%	90.11%	82.35%	0.28%	0.03%	0.16%	0.98%	93.43%	82.56%	-	-	-	1.45%	6.56%	12.14%
AR(1)EGARCH(1,2)	0.92%	71.72%	79.27%	0.28%	0.03%	0.16%	1.14%	54.33%	64.44%	1.53%	3.37%	2.09%	1.51%	4.08%	7.91%
AR(1)EGARCH(2,1)	1.03%	91.30%	80.80%	0.34%	0.12%	0.52%	1.09%	70.28%	73.77%	1.42%	8.70%	15.60%	1.67%	0.82%	1.76%

Table (10.4.A). Student-t Distribution, 500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.86%	54.53%	71.82%	0.45%	0.96%	3.34%	1.14%	54.33%	64.44%	-	-	-	-	-	-
AR(1)GARCH(0,2)	0.97%	90.11%	82.35%	0.51%	2.21%	6.92%	1.31%	20.56%	32.23%	-	-	-	-	-	-
AR(1)GARCH(1,1)	0.86%	54.53%	71.82%	0.23%	0.01%	0.04%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(1,2)	0.92%	71.72%	79.27%	-	-	-	0.87%	57.30%	73.46%	-	-	-	-	-	-
AR(1)GARCH(2,1)	0.86%	54.53%	71.82%	0.28%	0.03%	0.16%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	0.86%	54.53%	71.82%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.51%	2.21%	6.92%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.81%	39.40%	61.02%	0.68%	15.01%	32.48%	1.14%	54.33%	41.06%	-	-	-	-	-	-
AR(1)EGARCH(0,2)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.25%	29.39%	32.70%	-	-	-	-	-	-
AR(1)EGARCH(1,1)	1.03%	91.30%	80.80%	0.74%	24.14%	45.38%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(1,2)	1.24%	31.45%	44.59%	0.68%	15.01%	32.48%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(2,1)	1.19%	43.13%	38.66%	0.79%	36.26%	58.61%	-	-	-	-	-	-	-	-	-

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (11.1.A). GED, 2000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	9.17%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.67%	0.00%	0.00%	7.86%	0.00%	0.00%
AR(1)GARCH(0,2)	8.36%	0.00%	0.00%	5.89%	9.62%	0.65%	7.47%	0.00%	0.00%	7.39%	0.00%	0.00%	7.59%	0.00%	0.00%
AR(1)GARCH(1,1)	5.18%	72.25%	79.48%	6.17%	2.94%	8.36%	5.99%	5.77%	12.25%	6.08%	4.05%	11.45%	6.19%	2.28%	0.72%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	6.06%	4.85%	10.90%	5.94%	7.24%	18.34%	6.08%	4.05%	7.80%	6.25%	1.75%	0.13%
AR(1)GARCH(2,1)	5.23%	64.53%	74.32%	6.06%	4.85%	7.25%	-	-	-	5.97%	6.48%	16.75%	6.14%	2.94%	1.58%
AR(1)GARCH(2,2)	5.34%	50.30%	74.95%	6.06%	4.85%	7.25%	5.89%	9.00%	18.76%	6.08%	4.05%	10.16%	6.14%	2.94%	0.80%
AR(1)TARCH(0,1)	9.28%	0.00%	0.00%	6.23%	2.26%	0.00%	8.39%	0.00%	0.00%	7.50%	0.00%	0.00%	-	-	-
AR(1)TARCH(0,2)	-	-	-	6.00%	6.14%	0.36%	7.47%	0.00%	0.00%	7.17%	0.01%	0.03%	-	-	-
AR(1)TARCH(1,1)	5.88%	8.96%	12.43%	5.60%	25.37%	9.49%	5.61%	23.69%	33.29%	5.81%	12.34%	28.68%	6.14%	2.94%	8.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	5.77%	13.51%	18.92%	5.77%	14.55%	4.93%	5.83%	11.11%	22.81%	5.81%	12.34%	28.68%	6.14%	2.94%	6.49%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	9.39%	0.00%	0.00%	6.11%	3.79%	0.96%	8.83%	0.00%	0.00%	7.56%	0.00%	0.00%	8.24%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.69%	0.00%	0.00%	6.06%	4.85%	0.27%	8.12%	0.00%	0.00%	7.39%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(1)EGARCH(1,1)	6.75%	0.10%	0.27%	5.66%	21.26%	7.72%	6.65%	0.20%	0.36%	6.52%	0.44%	1.46%	6.79%	0.08%	0.33%
AR(1)EGARCH(1,2)	6.75%	0.10%	0.27%	5.72%	17.66%	26.92%	6.49%	0.52%	1.80%	6.46%	0.60%	1.00%	6.89%	0.04%	0.02%
AR(1)EGARCH(2,1)	6.80%	0.07%	0.19%	5.55%	30.02%	26.90%	6.54%	0.38%	1.29%	6.57%	0.32%	0.90%	6.89%	0.04%	0.06%

Table (11.2.A). GED, 1500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	8.47%	0.00%	0.00%	6.23%	2.26%	0.51%	8.56%	0.00%	0.00%	7.12%	0.01%	0.04%	7.43%	0.00%	0.00%
AR(1)GARCH(0,2)	8.15%	0.00%	0.00%	6.17%	2.94%	0.70%	7.47%	0.00%	0.00%	7.17%	0.01%	0.01%	7.49%	0.00%	0.00%
AR(1)GARCH(1,1)	5.61%	23.50%	32.48%	5.72%	17.66%	26.92%	6.21%	2.14%	6.03%	5.75%	15.04%	24.13%	5.92%	7.56%	4.43%
AR(1)GARCH(1,2)	5.88%	8.96%	21.94%	5.77%	14.55%	22.32%	6.05%	4.56%	12.18%	5.86%	10.04%	8.29%	6.14%	2.94%	0.15%
AR(1)GARCH(2,1)	5.61%	23.50%	32.48%	5.77%	14.55%	22.32%	-	-	-	5.64%	21.79%	38.84%	5.92%	7.56%	4.43%
AR(1)GARCH(2,2)	5.61%	23.50%	32.48%	5.83%	11.88%	18.27%	6.05%	4.56%	9.72%	5.75%	15.04%	24.13%	5.98%	6.05%	4.04%
AR(1)TARCH(0,1)	-	-	-	6.28%	1.72%	0.36%	8.45%	0.00%	0.00%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	6.17%	2.94%	0.70%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	6.26%	1.64%	3.38%	5.49%	35.23%	13.80%	5.89%	9.00%	12.76%	5.86%	10.04%	23.31%	6.19%	2.28%	2.46%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	6.21%	2.15%	4.46%	5.49%	35.23%	13.80%	-	-	-	-	-	-	6.09%	3.77%	5.30%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	8.80%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.12%	0.01%	0.02%	7.70%	0.00%	0.00%
AR(1)EGARCH(0,2)	7.99%	0.00%	0.00%	6.11%	3.79%	0.96%	7.79%	0.00%	0.00%	7.12%	0.01%	0.03%	7.59%	0.00%	0.00%
AR(1)EGARCH(1,1)	7.02%	0.02%	0.06%	5.55%	30.02%	11.52%	6.65%	0.20%	0.36%	6.24%	1.87%	5.58%	7.00%	0.02%	0.08%
AR(1)EGARCH(1,2)	6.85%	0.05%	0.21%	5.43%	40.99%	36.48%	6.65%	0.20%	0.56%	5.97%	6.48%	6.77%	6.84%	0.06%	0.08%
AR(1)EGARCH(2,1)	7.07%	0.01%	0.05%	5.55%	30.02%	11.52%	6.70%	0.14%	0.52%	6.35%	1.08%	2.23%	6.73%	0.11%	0.30%

Table (11.3.A). GED, 1000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	7.72%	0.00%	0.00%	5.89%	9.62%	3.00%	7.47%	0.00%	0.00%	6.63%	0.23%	0.70%	6.95%	0.03%	0.02%
AR(1)GARCH(0,2)	7.23%	0.00%	0.01%	5.77%	14.55%	12.56%	6.76%	0.10%	0.27%	6.41%	0.81%	2.38%	6.84%	0.06%	0.03%
AR(1)GARCH(1,1)	5.67%	19.70%	37.50%	5.72%	17.66%	26.92%	5.83%	11.11%	26.32%	5.91%	8.10%	16.58%	6.03%	4.80%	9.04%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	5.55%	30.02%	43.77%	5.72%	16.49%	35.94%	5.75%	15.04%	16.29%	5.92%	7.56%	4.43%
AR(1)GARCH(2,1)	5.67%	19.70%	37.50%	5.55%	30.02%	11.52%	-	-	-	-	-	-	5.92%	7.56%	16.24%
AR(1)GARCH(2,2)	5.83%	11.05%	22.43%	5.43%	40.99%	36.48%	5.78%	13.59%	30.95%	5.81%	12.34%	14.88%	5.87%	9.38%	4.80%
AR(1)TARCH(0,1)	-	-	-	5.83%	11.88%	3.87%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	5.55%	30.02%	11.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	5.38%	47.31%	19.08%	-	-	-	-	-	-	6.14%	2.94%	8.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	5.49%	35.23%	13.80%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	7.93%	0.00%	0.00%	5.72%	17.66%	6.21%	7.57%	0.00%	0.00%	6.68%	0.17%	0.40%	7.16%	0.01%	0.01%
AR(1)EGARCH(0,2)	7.61%	0.00%	0.00%	5.89%	9.62%	7.97%	6.98%	0.02%	0.00%	6.30%	1.43%	2.71%	7.00%	0.02%	0.01%
AR(1)EGARCH(1,1)	6.85%	0.05%	0.08%	5.72%	17.66%	6.21%	6.27%	1.64%	3.43%	5.86%	10.04%	13.42%	6.73%	0.11%	0.30%
AR(1)EGARCH(1,2)	7.02%	0.02%	0.07%	5.60%	25.37%	22.66%	6.27%	1.64%	4.66%	5.70%	18.18%	10.18%	6.52%	0.41%	0.52%
AR(1)EGARCH(2,1)	7.02%	0.02%	0.04%	5.66%	21.26%	7.72%	6.21%	2.14%	4.53%	5.86%	10.04%	8.29%	6.25%	1.75%	3.25%

Table (11.4.A). GED, 500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	6.37%	0.94%	3.16%	5.72%	17.66%	26.92%	6.70%	0.14%	0.24%	6.57%	0.32%	0.90%	6.68%	0.16%	0.05%
AR(1)GARCH(0,2)	6.31%	1.25%	3.53%	5.77%	14.55%	12.56%	6.16%	2.78%	7.67%	6.35%	1.08%	2.23%	6.41%	0.75%	0.42%
AR(1)GARCH(1,1)	5.45%	38.00%	62.79%	5.49%	35.23%	31.53%	5.78%	13.59%	28.98%	-	-	-	5.76%	14.09%	29.99%
AR(1)GARCH(1,2)	5.50%	32.64%	57.55%	5.26%	61.47%	52.22%	5.29%	57.73%	57.43%	-	-	-	5.71%	17.05%	26.68%
AR(1)GARCH(2,1)	5.45%	38.00%	62.79%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	5.45%	38.00%	62.79%	-	-	-	5.18%	72.92%	57.66%	-	-	-	5.65%	20.47%	38.15%
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	6.53%	0.38%	1.34%	6.00%	6.14%	9.31%	7.03%	0.02%	0.02%	6.52%	0.44%	1.46%	7.11%	0.01%	0.00%
AR(1)EGARCH(0,2)	6.26%	1.64%	4.23%	5.94%	7.72%	11.80%	6.43%	0.70%	1.13%	6.35%	1.08%	2.99%	6.84%	0.06%	0.03%
AR(1)EGARCH(1,1)	6.10%	3.57%	7.49%	6.06%	4.85%	3.67%	6.54%	0.38%	1.41%	6.24%	1.87%	5.91%	6.73%	0.11%	0.13%
AR(1)EGARCH(1,2)	6.42%	0.70%	2.45%	5.77%	14.55%	32.62%	6.27%	1.64%	5.02%	6.24%	1.87%	5.58%	6.73%	0.11%	0.13%
AR(1)EGARCH(2,1)	6.42%	0.70%	2.40%	5.60%	25.37%	22.66%	6.59%	0.28%	0.81%	6.57%	0.32%	0.90%	6.79%	0.08%	0.10%

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (12.1.A). GED, 2000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.37%	0.00%	0.00%	0.96%	87.19%	82.87%	2.67%	0.00%	0.00%	2.63%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)GARCH(0,2)	2.27%	0.00%	0.00%	1.02%	93.73%	81.98%	2.18%	0.00%	0.00%	2.14%	0.00%	0.01%	2.42%	0.00%	0.00%
AR(1)GARCH(1,1)	1.13%	57.23%	66.25%	0.85%	51.22%	70.34%	1.25%	29.39%	42.50%	1.53%	3.37%	7.75%	1.35%	15.43%	25.43%
AR(1)GARCH(1,2)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.25%	29.39%	42.50%	1.48%	5.50%	11.22%	1.40%	10.22%	17.93%
AR(1)GARCH(2,1)	1.24%	31.45%	44.59%	0.85%	51.22%	70.34%	-	-	-	1.53%	3.37%	7.75%	1.35%	15.43%	25.43%
AR(1)GARCH(2,2)	1.19%	43.13%	55.65%	0.85%	51.22%	70.34%	1.25%	29.39%	42.50%	1.53%	3.37%	7.75%	1.45%	6.56%	12.14%
AR(1)TARCH(0,1)	2.37%	0.00%	0.00%	0.96%	87.19%	82.87%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.13%	58.54%	67.75%	2.18%	0.00%	0.00%	2.14%	0.00%	0.01%	-	-	-
AR(1)TARCH(1,1)	1.24%	31.45%	44.59%	0.85%	51.22%	70.34%	1.14%	54.33%	64.44%	1.31%	19.78%	26.41%	1.18%	43.70%	56.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.20%	40.63%	53.58%	1.31%	19.78%	26.41%	1.24%	31.92%	45.06%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	2.37%	0.00%	0.00%	1.02%	93.73%	81.98%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.54%	0.00%	0.00%	1.19%	43.94%	56.92%	2.51%	0.00%	0.00%	2.52%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.67%	0.79%	2.41%	0.91%	68.49%	78.84%	1.53%	3.56%	7.01%	2.03%	0.01%	0.03%	1.83%	0.13%	0.29%
AR(1)EGARCH(1,2)	1.62%	1.40%	3.87%	0.91%	68.49%	78.84%	1.58%	2.12%	5.40%	1.75%	0.35%	0.43%	1.94%	0.03%	0.14%
AR(1)EGARCH(2,1)	1.73%	0.44%	1.45%	0.96%	87.19%	82.87%	1.58%	2.12%	5.40%	1.92%	0.05%	0.09%	1.88%	0.06%	0.15%

Table (12.2.A). GED, 1500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	0.96%	87.19%	82.87%	2.89%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)GARCH(0,2)	2.32%	0.00%	0.00%	0.91%	68.49%	78.84%	2.67%	0.00%	0.00%	2.25%	0.00%	0.00%	2.48%	0.00%	0.00%
AR(1)GARCH(1,1)	1.40%	10.02%	17.62%	0.68%	15.01%	32.48%	1.31%	20.56%	32.23%	1.48%	5.50%	11.22%	1.29%	22.56%	34.59%
AR(1)GARCH(1,2)	1.51%	3.98%	8.86%	0.62%	8.65%	21.35%	1.25%	29.39%	42.50%	1.48%	5.50%	11.22%	1.35%	15.43%	25.43%
AR(1)GARCH(2,1)	1.46%	6.41%	11.91%	0.68%	15.01%	32.48%	-	-	-	1.48%	5.50%	10.42%	1.29%	22.56%	34.59%
AR(1)GARCH(2,2)	1.51%	3.98%	8.86%	0.68%	15.01%	32.48%	1.31%	20.56%	27.08%	1.48%	5.50%	11.22%	1.35%	15.43%	25.43%
AR(1)TARCH(0,1)	-	-	-	0.96%	87.19%	82.87%	2.78%	0.00%	0.00%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.02%	93.73%	81.98%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	1.40%	10.02%	17.62%	0.74%	24.14%	45.38%	1.42%	9.12%	16.29%	1.64%	1.15%	3.28%	1.13%	57.88%	66.64%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	1.40%	10.02%	17.62%	0.74%	24.14%	45.38%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	2.21%	0.00%	0.00%	0.96%	87.19%	82.87%	2.72%	0.00%	0.00%	2.52%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.81%	0.00%	0.00%	1.25%	31.87%	45.51%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.78%	0.23%	0.84%	1.02%	93.73%	81.98%	1.85%	0.10%	0.24%	2.25%	0.00%	0.00%	1.99%	0.02%	0.04%
AR(1)EGARCH(1,2)	1.78%	0.23%	0.32%	0.96%	87.19%	82.87%	1.80%	0.20%	0.45%	1.97%	0.02%	0.05%	2.10%	0.00%	0.02%
AR(1)EGARCH(2,1)	1.83%	0.12%	0.47%	0.96%	87.19%	82.87%	1.85%	0.10%	0.24%	2.08%	0.01%	0.01%	2.10%	0.00%	0.02%

Table (12.3.A). GED, 1000 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	1.13%	58.54%	67.75%	2.29%	0.00%	0.00%	2.19%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(1)GARCH(0,2)	2.16%	0.00%	0.01%	1.08%	75.34%	76.58%	2.51%	0.00%	0.00%	2.14%	0.00%	0.01%	2.21%	0.00%	0.00%
AR(1)GARCH(1,1)	1.62%	1.40%	3.87%	0.74%	24.14%	45.38%	1.42%	9.12%	16.29%	1.48%	5.50%	10.42%	1.45%	6.56%	12.14%
AR(1)GARCH(1,2)	1.62%	1.40%	1.19%	0.74%	24.14%	45.38%	1.36%	13.92%	23.40%	1.53%	3.37%	6.67%	1.51%	4.08%	7.91%
AR(1)GARCH(2,1)	1.57%	2.39%	5.97%	0.68%	15.01%	32.48%	-	-	-	-	-	-	1.45%	6.56%	12.14%
AR(1)GARCH(2,2)	1.62%	1.40%	1.19%	0.74%	24.14%	45.38%	1.31%	20.56%	32.23%	1.53%	3.37%	6.67%	1.40%	10.22%	17.93%
AR(1)TARCH(0,1)	-	-	-	1.08%	75.34%	76.58%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.08%	75.34%	76.58%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.79%	36.26%	58.61%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.79%	36.26%	58.61%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	1.94%	0.03%	0.07%	1.19%	43.94%	37.70%	2.29%	0.00%	0.00%	2.08%	0.01%	0.00%	2.42%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.27%	0.00%	0.00%	1.25%	31.87%	45.51%	2.34%	0.00%	0.00%	2.14%	0.00%	0.01%	2.69%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.14%	1.08%	75.34%	76.58%	1.69%	0.69%	1.50%	1.97%	0.02%	0.10%	1.94%	0.03%	0.07%
AR(1)EGARCH(1,2)	2.16%	0.00%	0.00%	0.96%	87.19%	82.87%	1.69%	0.69%	1.50%	2.03%	0.01%	0.03%	1.94%	0.03%	0.07%
AR(1)EGARCH(2,1)	2.21%	0.00%	0.00%	1.08%	75.34%	76.58%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	2.15%	0.00%	0.00%

Table (12.4.A). GED, 500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	1.67%	0.79%	2.41%	1.13%	58.54%	67.75%	2.13%	0.00%	0.01%	2.25%	0.00%	0.00%	2.05%	0.01%	0.02%
AR(1)GARCH(0,2)	1.83%	0.12%	0.19%	1.08%	75.34%	76.58%	2.07%	0.01%	0.03%	1.92%	0.05%	0.09%	2.21%	0.00%	0.00%
AR(1)GARCH(1,1)	1.30%	22.19%	28.41%	0.91%	68.49%	78.84%	1.25%	29.39%	42.50%	-	-	-	1.51%	4.08%	7.91%
AR(1)GARCH(1,2)	1.57%	2.39%	1.71%	1.02%	93.73%	81.98%	1.31%	20.56%	32.23%	-	-	-	1.45%	6.56%	12.14%
AR(1)GARCH(2,1)	1.30%	22.19%	28.41%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	1.51%	3.98%	2.35%	-	-	-	1.31%	20.56%	32.23%	-	-	-	1.45%	6.56%	12.14%
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	1.67%	0.79%	2.41%	1.30%	22.33%	27.76%	2.13%	0.00%	0.01%	2.14%	0.00%	0.01%	2.42%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.05%	0.01%	0.03%	1.53%	3.85%	7.60%	2.23%	0.00%	0.00%	2.08%	0.01%	0.01%	2.58%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.78%	0.23%	0.32%	1.25%	31.87%	33.19%	1.69%	0.69%	1.50%	2.14%	0.00%	0.01%	2.32%	0.00%	0.00%
AR(1)EGARCH(1,2)	2.16%	0.00%	0.00%	1.19%	43.94%	56.92%	1.96%	0.03%	0.11%	2.30%	0.00%	0.00%	2.32%	0.00%	0.00%
AR(1)EGARCH(2,1)	1.89%	0.06%	0.11%	1.13%	58.54%	40.55%	1.74%	0.38%	0.84%	2.14%	0.00%	0.01%	2.48%	0.00%	0.00%

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)