

Between-seasons Competitive Balance in European Football: Review of Existing and Development of Specially Designed Indices

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Abstract

Competitive balance is a key issue for any professional sport league substantiated by its effect on demand for league games or other associated products. This work focuses on the measurement of between-seasons competitive balance, the longest time-wise dimension, which captures the relative quality of teams across seasons. The review of the existing indices examines their applicability in the context of European football. Given that domestic championships are multi-prized tournaments, as opposed to the more common North American unitary structure with a single prize, a set of specially designed indices that capture the complex structure of European football are introduced. An empirical investigation, using data from the English Premier League, further elucidates the main features of all appropriate indices by exploring their value and trend. It may be inferred that between-seasons competitive balance in England worsens through seasons mainly due to the very low ranking mobility of the top teams through seasons.

Keywords: competitive balance, sport economics, professional sport leagues, English Premier League, European football

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“The nature of the industry (of baseball) is such that competitors must be of approximate equal size if any are to be successful” (Rottenberg 1956).

1. Introduction

In his seminal article, Rottenberg (1956), one of the initiators of the economic analysis of sport, made an apt description of the concept of competitive balance when he argued that it is a unique attribute of professional team sports. The importance of competitive balance derives from the fact that it creates an uncertainty of outcome, which instigates the interest of sport fans leading to an increased demand for sport events (El-Hodiri and Quirk 1971; Rottenberg 1956). Since competitive balance is such an important concept for professional team sports, it has become a prominent topic of study in sports economics; yet, its quantification still remains an issue.

In our view, part of the problem is due to the way the quantification of competitive balance in professional team sports has been approached. According to Zimbalist (2003), the problem firstly arises from the fact that competitive balance is a multidimensional phenomenon. Therefore, a single index does not yet exist that can captures all its aspects. The present study focuses on the between-seasons competitive balance, which is the longest time-wise dimension and refers to the relative quality of teams across at least two consecutive seasons. From the fans’ perspective, it may be argued that a league balanced according to this dimension is preferable (Leeds and von Allmen 2008). Secondly, any optimal measure or index of competitive balance has to be important for fans and may differ from sport to another or even from one league to another (Zimbalist, 2003). This issue reflects the championship structure of a particular sport or league. In this study, we focus on football, which is the most popular professional team sport in the world (Reilly and Williams 2005). For instance, the FIFA World Cup final is rated as the biggest single-sport mega event in the world (Close 2010). The specific target of this study is the European professional football,

which is according to Gerrard (2004) “*the heartland of football, the only truly global team sport*”.

European football leagues are complex in structure, in that domestic championships are multi-levelled tournaments offering multiple prizes as opposed to the common single prize offered by North American ones (Kringstad and Gerrard 2007). This multi-levelled structure of European football has been further analysed by Manasis et al. (2013) who identify three levels in the championship tournaments in which teams compete for the corresponding ordered sets of prizes or punishments as follows:

- a) The first level refers to the competition for the championship title, which is considered the most prestigious prize in any league.
- b) The second level refers to the qualifying places for European tournaments the following season.
- c) Finally, the third level draws attention to the relegation places.

According to the work of Manasis et al. (2013) for the quantification of the seasonal dimension, the overall competitive balance is determined by the corresponding levels of competition involved in the pursuit of the various prizes offered¹. In particular, a new approach is offered on the premise that levels and ranking places should be rated according to their significance for fans. Evidently, the competition for the title is more important than that for relegation while a higher-ranking place is advantageous when participating in European tournaments. This multi-levelled structure of European football has so far not been considered when measuring the between-seasons dimension. Therefore, the aim of this study, following an analysis of the existing indices, is to develop a set of new indices for an enhanced quantification of competitive balance by employing the procedure offered by Manasis et al. (2013). An implementation and empirical investigation in English Premier League serves for an illustrative comparative analysis of all appropriate indices.

The article proceeds with Section 2 in which the existing indices are reviewed and examined in terms of their applicability in European football. The development of new specially designed indices in Section 3 is followed by the empirical investigation,

¹ The seasonal dimension refers to the relative quality of teams in the course of a particular season.

using data from the English Premier League since 1960, in Section 4. Concluding remarks derived from the analysis are offered in the final section of the article.

2. Review of existing indices

In contrast to the variety of existing seasonal competitive balance indices, the number of between-seasons indices presented in the literature is quite limited (Buzzacchi, Szymanski, and Valletti 2003). This number becomes even smaller when we focus on the implementation of such between-seasons indices on European football. This is justified by two important factors related to the structural features of measuring the between-seasons dimension:

(a) Teams' identity matters: In contrast to the closed North American leagues, European football leagues are open to new promoted teams substituting the worst teams of the previous season. Essentially, there are noticeable differences in the championship structure between a closed and an open league. In the former, identity of the teams remains exactly the same for a long period (except for seasons of expansion or contraction), whereas in the latter it continuously changes from season to season due to the promotion and relegation rule. More specifically, for every season in any domestic European football league, the last teams in the classification are demoted to the immediate lower division and are replaced by the promoted teams from the lower division. Consequently, even between two adjacent seasons, two, three, or even four teams change according to the specific relegation rule of the league.

Therefore, only indices that account for the promotion-relegation rule can be utilised for the study of competitive balance in European football. In an attempt to circumvent such a strict limitation, two different approaches emerged in the literature. In particular, Groot (2008) conveys the ranking of the relegated teams to the promoted ones while Gerrard (1998) reduces the total number of teams by excluding relegated teams. It is reasonable to assume that the former approach is preferable from the fans' perspective. Moreover, the latter excludes valuable information. In the present study the compromise proposed by Groot (2008) is followed, since it is assumed that it does not introduce an unacceptable degree of bias. This compromise cannot be applied for

a period longer than two adjacent seasons since the teams' identity in the league dramatically changes.

(b) The unit of measurement of the between-seasons indices: The two proposed units of measurement are: i) the *ranking mobility*, and ii) the *change in winning percentages/shares* across seasons. The former stands for relative performance while the latter for absolute level of success. It can be safely assumed that in the long run, relative performance is more significant than the absolute level of success. Obviously, the change in the teams' winning percentages across seasons matters to the fans, but it is doubtful that this is at least equally important as ranking mobility. Normally, fans cannot easily judge teams' winning percentages from season to season. On the contrary, they can spontaneously recall at least the approximate ranking position of all teams. In particular, they can easily recall the exact position of teams at the top of the ladder in the span of one or even two and/or three seasons.

Following that, indices of ranking mobility across two adjacent seasons, although they do not account for the promotion-relegation rule, they can be applied to European football under the above-mentioned compromise. However, the same rationale cannot be followed for the indices of winning percentages change. While it seems natural to assign the ranking of the relegated to the promoted teams, a similar procedure for the winning percentages appears to be quite arbitrary. Consequently, indices based on the winning percentage change cannot be applied on European football data.

2.1. Indices Appropriate for European Football

The number of between-seasons indices applicable to European football is quite limited, mainly due to the implications generated by the promotion-relegation rule. In the following, we present the G index along with three mostly statistical indices that can be applied on European football using the above-discussed compromise. More specifically, using t as a benchmark season, promoted teams in season $t-1$ are assigned to the ranking position of the relegated ones. The exact ranking order of the promoted teams is determined by the respective ranking position in the lower division in season $t-1$. Lastly, for comparability issues, an appropriate modification to the conventional range from zero (perfect balance) to one (complete imbalance) is attempted.

The G index

An index especially designed for European football is the so called G index (G) (Buzzacchi, Szymanski, and Valletti 2003). Essentially, the G index, not only accounts for the promotion and relegation rule, but it also permits for a comparison across leagues and/or seasons with various number of teams. Additionally, it accounts for the number of teams promoted in and relegated from any division in a particular championship format. It is a *Gini type* index which measures the cumulative frequency of teams entering the top K positions in the highest league over a fixed period. Moreover, it measures the turnover in the top K positions relative to the expected frequency in a perfectly balanced league in which the win in every game is purely random. Buzzacchi, Szymanski, and Valletti (2003) compare the observed frequency with a theoretical benchmark which represents the number of teams entering the top K places in an ideally balanced league. The elaborated benchmark considers a typical European championship format with a number of L divisions, where $p(l)$ teams are promoted and $r(l)$ teams are relegated each season in league l with N_l teams. Under the assumption of competitive balance, the probability that a team is in division l in year t is given by:

$$d(l,t) = d(l,t-1) \frac{N_l - r(l) - p(l)}{N_l} + d(l-1,t-1) \frac{r(l-1)}{N_{l-1}} + d(l+1,t-1) \frac{p(l+1)}{N_{l+1}}, \quad (1)$$

where $1 \leq l \leq L$, $r(L) = p(1) = 0$, $d(0,t) = d(L+1,t) = 0$. The starting year is 0, t is any year in the period under examination T . Each team starts at $t=0$ in league l with probability 1, consequently $d(l,0) = 1$ and 0 otherwise. Given that the probability a team that started (at $t=0$) from league j to be placed in one of the top K positions in the highest league in year t is estimated by the joint probability $d_j(l,t)K/N_l$, the probability that the same team is at least once in any of the top K positions after year T is given by:

$$w_l(K,T) = 1 - \prod_{t=0}^{T-1} \left[1 - \frac{d_l(l,t)K}{N_l} \right]. \quad (2)$$

Based on equation (2), the expected number of teams that will have been in any of the top K places after T years is given by:

$$y^L(K, T) = \sum_{l=1}^L N_l w_l(K, T). \quad (3)$$

The G index is proposed by Buzzacchi, Szymanski, and Valletti (2003) after calculating the benchmark case in (3); the index quantifies the observed values as:

$$G(T) = \frac{\sum_{t=1}^T y^L(K, t) - \sum_{t=1}^T y_a^L(K, t)}{\sum_{t=1}^T y^L(K, t)}, \quad (4)$$

where T stands for the years under consideration and $y_a^L(K, T)$ stands for the observed number of teams entering at least once in the top K positions in the highest league. The lower bound of G is well defined as it equals zero and it is obtained in the case of perfect balance. Theoretically, G could take negative values if the observed $y_a^L(K, T)$ number is larger than the expected $y^L(K, T)$ number of teams. However, the upper bound (G_u) of the index, which indicates a completely unbalanced league, is not well defined and is only referred to as “close to one”. Therefore, a modification is required for the proper application of G to European football.

In fact, the value of G_u , is always lower than one. That can be easily derived from (4), in which the nominator is smaller than the denominator. It is important to point out that the minimum value of the observed number $y_a^L(K, T)$ is always K regardless of T . In effect, this stands for the case of a completely unbalanced league in which the top K teams dominate the league over a period of T seasons. Intuitively, K comprises another benchmark which has to be taken into consideration when calculating G . Therefore, for comparability issues, we propose the *Adjusted G Index* (aG) given by:

$$aG = \frac{\sum_{t=1}^T y^L(K, t) - \sum_{t=1}^T y_a^L(K, t)}{\sum_{t=1}^T y^L(K, t) - K}. \quad (5)$$

The value of aG ranges from zero (perfect balance) to one (complete imbalance). However, the main attribute of aG is that it provides better estimation in cases close to complete imbalance which is our main concern. For illustration purposes, consider a

closed league in which four teams enter the top three places over a period of ten years². The calculation of both G and aG is presented in Table 1 for some realistic values of N . It can be easily derived that the calculation differs substantially between the two indices. Moreover, G over-estimates the level of competitive balance in comparison with aG . In particular, the value of aG is close to complete imbalance (from 0.851 to 0.928), whereas G offers lower and a wider range of values (from 0.588 to 0.764). It must be noted, the difference between the two indices is higher for small values of N .

Table 1: Calculation of G and aG for $\sum_{t=1}^T y_a^L(K, t) = 4$, $T=10$ and $K=3$

N	G	aG
10	0.588	0.851
12	0.647	0.880
14	0.686	0.897
16	0.714	0.909
18	0.735	0.917
20	0.751	0.923
22	0.764	0.928

Index of Dynamics

Haan, Koning, and van Witteloostuijn (2002) propose the *index of Dynamics* (DN_t) to measure ranking mobility from season to season by summation of the absolute number of ranking changes of all teams. Consequently, the mathematical expression of DN_t is given by:

$$DN_t = \sum_{i=1}^N |r_{i,t} - r_{i,t-1}|, \quad (6)$$

where $r_{i,t}$ stands for the ranking position of team i in year t . As it is illustrated in the following example, DN_t is a quite simple index, which can be calculated in a straightforward manner. Consider a six-team league and the final rankings in two consecutive seasons denoted as A and B .

² A closed league is selected only for the sake of simplicity. However, the same conclusions can also be drawn for open leagues.

Table 2: A six-team League: Ranking Changes

Teams	<i>season A</i>	<i>season B</i>	Change
A	1	6	5
B	2	5	3
C	3	4	1
D	4	3	1
E	5	2	3
F	6	1	5
<i>Sum of Change:</i>			18

As it can be inferred from the above example, upward and downward movements in the rankings are treated identically. In addition, the summation of change in rankings is affected by the number of N teams. If the number of teams is N , the maximum of DN_t equals $N^2/2$. However, in view of the fact that DN_t depends on the number of teams that comprise the league, Haan, Koning, and van Witteloostuijn (2002) introduce the normalised *Index of Dynamics* (DN_t^*) in league rankings³:

$$DN_t^* = \frac{2}{N^2} \sum_{i=1}^N |r_{i,t} - r_{i,t-1}|. \quad (7)$$

The DN_t^* index is insensitive to N and the lower bound of zero stands for a completely unbalanced league (no ranking mobility) while the upper bound of one stands for the case of a perfectly balanced league (maximum ranking mobility). In the above example, the value of DN_t^* equals one, since it reaches the maximum ranking mobility from season A to season B. Following equation (7), for comparability issues, the new formula of DN_t^* index is given by:

$$DN_t^* = 1 - \frac{2}{N^2} \sum_{i=1}^N |r_{i,t} - r_{i,t-1}|. \quad (8)$$

Based on equation (8), the range of DN_t^* is conventionally defined from zero (maximum ranking mobility) to one (no ranking mobility). The former is obtained in

³ Even though they do not refer to Haan, Koning, and van Witteloostuijn (2002), Mizak, Neral, and Stair (2007) propose the *Adjusted Churn*, which is fundamentally the same as DN_t^* .

the case of a dynamically perfectly balanced league, whereas the latter in that of a dynamically completely unbalanced league.

Kendall's tau coefficient

Groot (2008) introduces the application of the *Kendall's tau coefficient* (τ) to rank correlation. The τ index illustrates the overall ranking turnover within a league between two seasons. The calculation of τ is based on the number of transpositions required to transform a particular rank order to another specific order. For example, suppose the following ranking in a league with four teams:

	Teams			
	A	B	C	D
<i>season A:</i>	1	2	3	4
<i>season B:</i>	3	1	2	4

Note that when teams are orderly listed in *season A*, two transpositions are required to transform the ranking in *season A* into the ranking of *season B*. More specifically, team *C* in *season A* has to advance two positions in *season B*. The number of observed transpositions (s) is the basis for the calculation of the τ index. In essence, s is compared with the maximum possible transpositions (s_{max}), which is equal to $N(N-1)/2$. The formula of the τ index is given by:

$$\tau = 1 - \frac{2s}{s_{max}} = 1 - \frac{4s}{N(N-1)}. \tag{9}$$

The theoretical upper and lower bounds of this statistical index are -1 and 1. For an effective comparison among indices, a modification via rescaling is attempted as follows:

$$\tau = \frac{1 + \left[1 - \frac{4s}{N(N-1)} \right]}{2} = 1 - \frac{2s}{N(N-1)}. \tag{10}$$

The new range of the index is from zero to one, which stands for the cases of a dynamically perfectly balanced and a dynamically completely unbalanced league respectively. The former is defined by the maximum number of transpositions while the latter by the absence of transpositions from season to season. Using this

transformation, the behaviour of the index can be effectively contrasted with the remaining indices of competitive balance.

Spearman's rho

A competitor to *Kendall's tau* is *Spearman's rho* (r_s) correlation coefficient for ranked data (Maxcy and Mondello 2006; Maxcy 2002; Daly and Moore 1981). Although Kendall bases his statistic on the number of inversions or ranking transpositions, Spearman treats ranks as scores and then calculates the correlation between two sets of ranks. Based on the formula given by Snedacor and Cochran (1967):

$$r_s = 1 - \frac{6 \sum_{i=1}^N D_i^2}{N(N^2 - 1)}, \quad (11)$$

where $D_i=(X_i-Y_i)$ stands for the difference in rankings of teams between the two seasons. The interpretation of r_s is similar to that of the τ index, and its value ranges from -1 (perfect balance) to 1 (complete imbalance). For comparability issues, a similar rescaling with this in τ is attempted for r_s as follows:

$$r_s = \frac{1 + \left[1 - \frac{6 \sum_{i=1}^N D^2}{N(N^2 - 1)} \right]}{2} = 1 - \frac{3 \sum_{i=1}^N D^2}{N(N^2 - 1)}. \quad (12)$$

The new range of the index is from zero to one, which stands for the cases of a dynamically perfectly balanced and a dynamically completely unbalanced league respectively. The former is defined by the maximum ranking difference while the latter by the absence of ranking difference from season to season.

2.2. Indices not Applicable to European Football

In this section, we present indices that cannot be applied to European football due to the promotion-relegation rule. Generally, their distinguishing feature is that their unit of measurement is based on the ranking over long periods (more than two) or winning percentage change across seasons. In both cases, the compromise adopted previously cannot be followed. Our review, therefore, focuses only on the main features of those indices.

2.2.1. Indices of Ranking over Long Periods

There exist indices of ranking mobility that refer to a much longer period than that of two adjacent seasons. Apparently, for a period of many seasons, the teams' identity in the league dramatically changes and, therefore, the compromise to overlook the promotion-relegation rule cannot be applied. For instance, in a three-season span for a league with 18 teams in total and 3 relegated teams, the change in the teams' identity could rise up to 50 percent; as a result, those indices of ranking mobility cannot be applied to European football.

Hirfindahl-Hirschman Index

There are two widely used applications of the *Hirfindahl-Hirschman Index (HHI)* for the measurement of the between-seasons dimension. Firstly, the *relative Hirfindahl-Hirschman Index (rHHI)*, which measures concentration of title winners or other top places over a long period of time (Eckard 1998). Secondly, the *Hirfindahl-Hirschman Index adjusted (HHI-adj)*, which was introduced by Gerrard (2004). Although *HHI-adj* has a similar application to *rHHI*, it takes into account the value of *HHI* under domination by the same teams for the whole examined period.

Gini Coefficient

Besides *HHI*, *Gini Coefficient (Gini)* has also been employed for measuring the concentration in any of the top positions in a league over a period of many seasons (Fizel 1997; Quirk and Fort 1997).

Markov-based Approach

Hadley, Cieka, and Krautman (2005) introduce a *Markov-based approach* to estimate transitional probabilities of teams from one state to another over a period of two decades whereas Krautmann and Hadley (2006) employ this approach clustering a number of seasons specified by structural factors in Major League Baseball (MLB).

Hope Statistic

Similarly to the *Markov-based approach*, the *Hope Statistic*, introduced by Kaplan, Nadeau, and O'Reilly (2011), handles success as a binary variable. Instead of using winning percentages, the *Hope statistic* employs a chosen number of wins out of a specified ranking spot as an indicator of hope. The chosen number of wins is quite

arbitrary. For instance, Kaplan, Nadeau, and O'Reilly (2011) use the number of 8 wins while O'Reilly et al. (2008) use 5.5 wins away from the post-season spot.

2.2.2. Indices of Winning Percentage Chance Across Seasons

A number of indices that measure winning percentage/share change of teams across seasons exist. In what follows, we briefly review the most important of those indices mainly employed in closed leagues either in the United States or in Australia.

Correlation Coefficient

The *Correlation coefficient* of teams' winning percentages up to a three seasons lag was utilised by Balfour and Porter (1991) to investigate the effects of free agency in competitive balance both in MLB and the National Football League (NFL). Similarly, Butler (1995), employs the *Correlation* of winning percentages for an adjacent season for the analysis of competitive balance in MLB.

ANOVA-based indices

The *ANOVA-based measure (VAR)*, developed by Eckard (1998; 2001; 2001), and the *Competitive Balance Ratio (CBR)*, introduced by Humphreys (2002), are more sophisticated measures that encompass both the seasonal and the between-seasons dimensions. Both *VAR* and *CBR* are calculated over a period of several seasons; what is more, there is some controversy over their resemblance (Humphreys 2003; Eckard 2003). An index similar to the *VAR* and *CBR* spirit is James's index which is measured for a decade and is also composed of two elements (James 2003).

Linearised Turnover Gain Function

Lastly, the *Linearised Turnover Gain Function (LTFG)*, was recently introduced by Lenten (2009). It uses the winning percentages of two consecutive seasons to produce a quadratic metric that takes the form of a turnover gain function.

3. Development of Specially Designed Indices

Based on the preceding review, a number of different approaches have been used for the development of the existing indices. However, none of them takes into account the characteristics of the complex structure in European football leagues. The objective here is to develop special indices of between-seasons competitive balance by rating levels and ranking positions according to their significance from fans' perspective. For the construction of the new indices the above reviewed *Index of Dynamics (DN_t^{*})*

is employed. Essentially, DN_t^* index is calculated by equally rating ranking places. However, the relative significance of the various levels and/or ranking positions in European football is not the same; and thus, they have to be rated accordingly. Following the procedure for the seasonal dimension in Manasis et al. (2013) using the NCR_K , a proper adjustment of DN_t^* is necessary to effectively capture the three levels of competitiveness which lead to different prizes-goals.

3.1. Dynamic Index

The *Dynamic Index* (DN_K) is analogous to the NCR_K index in the seasonal dimension; thus, it can be interpreted as the degree of dynamic domination by the top K teams. Following the procedure for DN_t^* in equation (8), for the proper design of DN_K , it is necessary to identify the maximum ranking mobility for the top K teams ($maxDN_K$), which is reached when the top K teams are the ones ended at the bottom K places of the previous season. To illustrate, consider a league which exhibits maximum ranking mobility, that is, an inverse ranking order from season to season, as is shown in Table 3. It should be emphasized that the maximum ranking mobility stands for a dynamically perfectly balanced league. In that case, the ranking difference for the first team equals $N-1$, for the second team $N-3$, and so on down to the middle of the ladder. The absolute ranking difference for the bottom half of the ladder is identical as far as the reverse order is concerned.

Table 3: Maximum Ranking Mobility

r in season t	r in season $t-1$	$ r_{i,t} - r_{i,t-1} $
1	N	$N-1$
2	$N-1$	$N-3$
3	$N-2$	$N-5$
4	$N-3$	
...		...
i	$N-(i-1)$	$N-(2i-1)$
...		
$N-3$	4	
$N-2$	3	$N-5$
$N-1$	2	$N-3$
N	1	$N-1$
<i>Total:</i>		$\frac{N^2}{2}$

Hence, the maximum absolute ranking change for the i th team equals $N-(2i-1)$ and the $\max DN_K$ is given by:

$$\max DN_K = \sum_{i=1}^K |r_{i,t} - r_{i,t-1}| = \sum_{i=1}^K (N - (2i - 1)) = K(N - K), \text{ for } K \leq N/2. \quad (13)$$

Following the procedure in equation (8), DN_K is given by:

$$DN_K = 1 - \frac{\sum_{i=1}^K |r_{i,t} - r_{i,t-1}|}{\max DN_K} = 1 - \frac{\sum_{i=1}^K |r_{i,t} - r_{i,t-1}|}{K(N - K)} = 1 - \frac{\sum_{i=1}^K r_i}{K(N - K)}, \text{ for } K \leq N/2, \quad (14)$$

where $r_{i,t}$ and r_i stand for the ranking position of team i in season t and for the absolute ranking difference of the i th team from season $t-1$ to season t , respectively. The index ranges from zero (maximum ranking mobility by the top K teams) to one (no ranking mobility by the top K teams). The former stands for absence of dynamic domination, which is reached when the top K teams are derived from the bottom K places of the previous season. As far as the latter is concerned, it stands for a completely dynamically dominated league, which is obtained when the ranking position of the top K teams remains unchanged across two adjacent seasons. As DN_K increases, the mobility of the top K teams decreases and, thus, they become more dynamically dominant. A major advantage of this index is that it can be used for the study of competitive balance across leagues with various N .

3.2. Dynamic Index for the Champion

For $K=1$, it can be easily derived that DN_1 captures the first level and it can be interpreted as the degree of the champion's (the first team's) ranking mobility. Following equation (14), the *Dynamic Index for the Champion* (DN_1) is given by:

$$DN_1 = 1 - \frac{|r_{1,t} - r_{1,t-1}|}{\max DN_1} = 1 - \frac{r_1}{(N - 1)}. \quad (15)$$

The lower bound of zero is obtained in the case of maximum ranking mobility, which is interpreted as the absence of dynamic domination in a league by the champion; that is, the champion comes from the last ranking place of the previous season. As far as

the upper bound one is concerned, it is obtained in the case of no ranking mobility, which is interpreted as a league which is completely dynamically dominated by the champion; that is, the champion wins the championship for two consecutive seasons. The higher the DN_1 , the more dynamically dominant the champion becomes.

3.3. Adjusted Dynamic Index

The *Adjusted Dynamic Index* (ADN_K) is now introduced as a natural development of DN_K . This index captures both the first and the second levels in the multi-prized tournament structure of European football. The design of the index will be illustrated using a simple example of a 10-team league with two teams participating in the European tournaments. In that case, the champion stands for the first level while the second ranking team stands for the second level. Although DN_1 effectively demonstrates the mobility in the first level, DN_2 alone cannot capture each of the levels, since it rates them equally. Thus, the development of an index which accounts for the relative importance of each level would be very beneficial for the measurement of competitive balance across seasons. An average index effectively captures the relative significance since it adjusts for the relative mobility of each level as it is presented in Table 5. The leagues in *seasons A* and *B* display identical cumulative absolute ranking change for the 1st and 2nd team. However, the specific ranking position of the first two teams markedly differs from *season A* to *season B*. DN_1 and DN_2 effectively demonstrate the degree of mobility or dynamic domination by the champion and the top two teams respectively. However, DN_2 fails to account for the relative importance of the two ranking places, or else to capture the ranking mobility between the two teams. Arguably, *season B* is more balanced than *season A*, although this cannot be captured by DN_2 . For that reason, the average of the two indices is employed for an enhanced quantification of competitive balance.

Table 4: Average of DN_1 & DN_2

<i>Starting season (S)</i>	<i>season A</i>	$ r_{i,S} - r_{i,A} $	<i>season B</i>	$ r_{i,S} - r_{i,B} $
1	3		3	
2	4		4	
3	1	2	2	1
4	2	2	1	3
5	5		5	
6	6		6	
7	7		7	
8	8		8	
9	9		9	
10	10		10	
	DN_1:	0.777		0.666
	DN_2:	0.75		0.75
	Average (DN_1, DN_2):	0.763		0.708

In essence, the resultant average index captures both levels and rates them accordingly. This procedure can be generalised for any number of the top K positions as long as their value is unequally rated. Thus, the ADN_K is derived by adjusting for the relative significance of the top K positions. Following the procedure in Manasis et al. (2013) for the corresponding ACR_K index along with the formula for DN_K in equation (14), ADN_K is given by:

$$ADN_K = \frac{\sum_{i=1}^K DN_i}{K} = 1 - \frac{2}{K} \left[\sum_{i=1}^K w_i r_i \right], \text{ for } K \leq N/2, \quad (16)$$

where w_i stands for the weight attached to the i th team and derived from the partial sum of the harmonic series with first term $1/[2(N-1)]$ and last term $1/[2K(N-K)]$. The weights w_i are identical to the corresponding ones used to define the seasonal ACR_K index; for details, see equations 5 and 6 in Manasis et al. (2013). The range of ADN_K accords with the conventional zero to one. The lower bound holds both for absence of dynamic domination by the top K teams and perfect dynamic competition among the same teams. The lower bound is obtained in the case of maximum ranking mobility in the reverse order; that is, the top K teams inversely come from the bottom of the ladder of the previous season. As the index increases, the mobility of the top K teams decreases and, thus, they become more dynamically dominant. On the other hand, the upper bound stands for a dynamically completely dominated league by the top K

teams and absence of dynamic competition among the same teams. The upper bound is obtained when there is no ranking mobility in the top K teams. Since the range of the component indices are insensitive to the values in N and K as previously described, ADN_K also has a similar behaviour. The ADN_K is interpreted as:

- a) The degree of ranking mobility or dynamic domination by the top K teams.
- b) The degree of ranking mobility or dynamic competition among the top K teams.

The two distinguishing features of ADN_K , are as follows:

- a) It can be decomposed into its K component indices; thus, the ingredient sources of dynamic domination can be determined.
- b) It rates the top K ranking positions at a decreasing function of their ranking position according to the criteria set in Section 1. The discussion for the sensitivity of w_i to K , N , and ranking position i in ACR_K also holds for the ADN_K index (see Manasis et al. 2013, 369).

3.4. Dynamic Index for Relegated Teams

As already discussed, the promotion-relegation rule is characteristic in European football structure. For this reason we introduce the *Dynamic Index for Relegated Teams* (DN^I) that captures the degree of ranking mobility of the I relegated teams or the degree of dynamic competition for relegation. According to Table 4, in which the league exhibits the maximum ranking mobility, the absolute ranking change at the bottom is similar to that at the top of the ladder. As a result, the maximum ranking mobility of the I relegated teams ($\max DN^I$) is given by:

$$\max DN^I = \sum_{i=N-I}^N |r_{i,t} - r_{i,t-1}| = \sum_{i=1}^I (N - (2i - 1)) = I(N - I), \text{ for } I \leq N/2. \quad (17)$$

Therefore, the DN^I index is given by:

$$DN^I = 1 - \frac{\sum_{i=N-I+1}^N |r_{i,t} - r_{i,t-1}|}{I(N - I)} = 1 - \frac{\sum_{i=N-I+1}^N r_i}{I(N - I)}, \text{ for } I \leq N/2. \quad (18)$$

The range of the index is from zero to one. The former stands for the maximum ranking mobility while the latter stands for absence of ranking mobility. DN^I does not account for the ranking mobility among the I teams. Consequently, the reverse order

is not required for the maximum ranking mobility of the I relegated teams. A major advantage of the index is that it provides a reliable estimation for the ranking mobility of the I relegated teams regardless of the variation in N and/or I . Hence, DN^I can be adjusted according to the specific promotion-relegation rule and can be used for an analysis of competitive balance across leagues and/or seasons with variant N . A limitation of the index is that it does not provide any information for the ranking mobility of each particular demoted team which is of limited importance for fans.

3.5. Special Dynamic Index

Lastly, the *Special Dynamic Index* (SDN_K^I) is introduced to account for all three important levels in the multi-prized European football leagues. SDN_K^I is a composite index, since a number of simpler indices are employed for its design. Based on the approach followed by Manasis et al. (2013) for the seasonal corresponding SCR_K^I index and the formulas for the ADN_K (16) and DN^I (18) indices, the function of SDN_K^I is given by:

$$SDN_K^I = \frac{\sum_{i=1}^K DN_i + DN^I}{K+1} = 1 - \frac{2}{K+1} \left[\sum_{i=1}^K w_i r_i + \sum_{i=N-I+1}^N w_I r_i \right], \text{ for } I \leq N/2, K \leq N/2, I+K < N, \quad (19)$$

where the weight w_i for the top K teams is identical to this in the corresponding seasonal SCR_K^I index, and the weight w_I attached to bottom I teams is given by:

$$w_I = \frac{1}{2I(N-I)}, \text{ for } I \leq N/2 \quad (20)$$

The zero value (lower bound) of the SDN_K^I is reached for the maximum ranking mobility among the top K teams as well as for the maximum ranking mobility of both the top K and the bottom I teams. Essentially, the top K teams inversely come from the bottom K positions, whereas the I relegated teams come from the top I positions of the previous season. The value of one (upper bound) is reached when no ranking mobility is observed in both the top K and the bottom I positions. The range of SDN_K^I is insensitive to values of N , K , and I making comparisons between different league formats feasible. The interpretation of this composite index is specified by three

different qualities: (a) the degree of ranking mobility of the top K teams, (b) the degree of ranking mobility among the top K teams, and (c) the degree of ranking mobility of the I relegated teams. The innovative features of the index regarding weight attached to the top K and bottom I teams are depicted and discussed in Manasis et al. (2013, 372-373).

4. Empirical Investigation

An empirical investigation, using real data from Premier League (from 1959/60 to 2012/13 seasons), may further elucidate the key points by exploring the value and the trend of the introduced as well as existing applicable indices. A detailed comparison of all indices behaviour is attempted via graphical presentation and correlation analysis.

From the descriptive statistics presented in Table 5, what is a cause for concern is the fact that indices reach values close to complete imbalance (values higher than 0.5 and close to unity) which is of particular concern. This may interpreted as low ranking mobility across seasons. In particular, DN_1 displays the highest while DN_t^* displays the lowest mean value. In particular, DN_1 is very close to its upper bound (0.856) suggesting that the relative mobility of the champion from season to season is very small in England. Therefore, what also causes concern is the champion's dynamic domination or the tendency to remain in the first place for two adjacent seasons. On the other hand, the lower values in DN^l index, signifies that the promotion-relegation rule greatly contributes to a more competitive championship and, thus, proves to be a useful mechanism in English football.

Table 5: Between-seasons Indices for Premier League

Index	Mean	SD	Min	Q ₁	Median	Q ₃	Max
τ	0.698	0.074	0.563	0.645	0.697	0.749	0.856
r_s	0.767	0.084	0.589	0.704	0.777	0.824	0.909
DN_t^*	0.577	0.086	0.421	0.521	0.580	0.645	0.740
DN_1	0.856	0.211	0.000	0.857	0.947	0.952	1.000
DN^l	0.731	0.133	0.350	0.667	0.765	0.824	1.000
ADN_K	0.801	0.114	0.447	0.753	0.817	0.889	0.972
SDN_K^l	0.790	0.106	0.468	0.753	0.808	0.861	0.930
aG	0.588	0.172	0.255	0.469	0.560	0.759	0.925

It is interesting to note that the range of DN_1 reaches its maximum attainability. In particular, the range of DN_1 equals unity, since the best and worst records are equal to the lower and upper bounds of the index respectively. The appearance of the upper bound (unity) for DN_1 is justified, since it is reached when a single team wins the championship for two consecutive seasons. On the other hand, the lower bound (zero) DN_1 is achieved when the last promoted team wins the championship the following season, which is a quite infrequent incident⁴. For the entire investigated period, two promoted teams won the championship. More specifically, the first case concerns the remarkable 1977 season, during which Nottingham Forest (the third out of three promoted teams) won the league title while the second case concerns the 1961 season, during which Ipswich (first promoted team) also won the championship title.

The trend pattern and fluctuation of the indices is effectively depicted by the moving average (MA) for five seasons time series. For illustration purposes, following Kamerchen and Lam (1975), the indices are classified as partial and summary ones⁵. In Figure 1, an almost identical pattern is noted among the three summary indices, which is an indication of strong correlation. More importantly, in Figure 2, the extremely high values of most partial indices are illustrated, which is indicative of a considerably unbalanced league across seasons. In that graph, a remarkably decline and values close to complete imbalance is noticed for most of the indices after the mid of 90's. For this decline, the explanation given by Mitchie and Oughton (2004) for a growing gap between the top teams and the rest due to the increased revenue sources for successful results is adopted. By investigating the behavior of the component indices, it can be further explored the ingredient sources of competitive balance.

⁴ It should be reminded that the promoted teams are orderly assigned the ranking place of the relegated teams.

⁵ A partial index provides information for a few teams, whereas a summary index provides information for all the teams that make up the league.

Figure 1: Moving Averages of lag five for Summary Indices in Premier League from 1959/60 to 2012/13
(Dynamic Index, Spearman and Kendall Correlations)

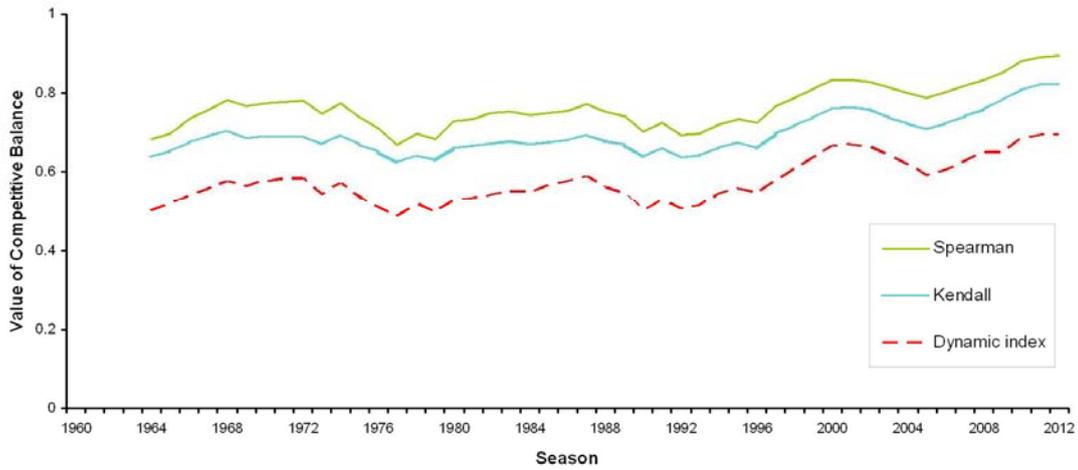
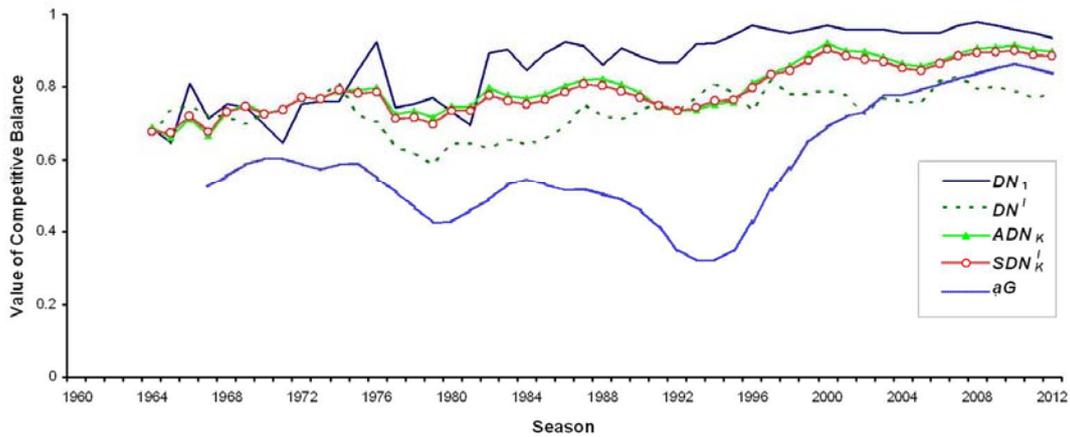


Figure 2: Moving Averages of lag five for Partial Indices in Premier League from 1959/60 to 2012/13
(DN_1 , DN^I , ADN_K , SDN_K^I , aG)



A growing gap between the DN^I and the rest of the new indices should be emphasised. Based on the properties of those indices, it may be drawn that there is a greater diversity in the identity of the relegated teams as compared to that of the top teams across seasons. We can observe that the lower the ranking position, the greater the dynamic competition. Alternatively, the promotion-relegation rule contributes more than the champion to a dynamically balanced championship.

As expected, a similar upward linear trend is found for the three summary indices based on the trend analysis results in Table 6. Interesting enough, DN_1 exhibits not only the highest values but also the strongest upward trend, which is interpreted as a significant dynamic domination by the champion. For instance, Manchester United

won the championship title in 13 out of the 22 last seasons. To put the trend in perspective, the total increase for the entire period based on the most comprehensive SDN_K^I index (which captures all three levels) rises up to 30%. This is interpreted as a serious deterioration of between-seasons competitive balance in the course of the seasons. Trend of second degree is found for the aG index which measures the mobility in the top five positions in a five-season span. Note the low levels of aG in the mid of 90's in contrast to the recent high values. Consequently, the worsening of competitive balance in England during the last decade may be explained by the very low ranking mobility of the top five teams across seasons. For instance, starting from season 1991, the number of teams appeared in the top five places the last five seasons was 15, whereas this number was six & eight for the 2008 and 2012 seasons respectively.

Table 6: Coefficients of the Trend Regression Model

Index	C		T		T ²
τ	0.623 ^{***}	(0.017)	0.0028 ^{***}	(0.0005)	
r_s	0.685 ^{***}	(0.020)	0.0030 ^{***}	(0.0006)	
DN_t^*	0.490 ^{***}	(0.019)	0.0032 ^{***}	(0.0006)	
DN_1	0.688 ^{***}	(0.052)	0.0061 ^{***}	(0.0017)	
DN^I	0.654 ^{***}	(0.035)	0.0028 ^{***}	(0.0011)	
ADN_K	0.689 ^{***}	(0.026)	0.0041 ^{***}	(0.0008)	
SDN_K^I	0.678 ^{***}	(0.023)	0.0041 ^{***}	(0.0007)	
aG	0.672 ^{***}	(0.064)	-0.0193 ^{***}	(0.0051)	0.0004 ^{***} (0.0001)

Trend T is tested via ordinary least squares up to third degree. The numbers in parentheses are standard errors; C is the constant of the regression.

^{***}Significant at $\alpha=1\%$.

The next step of the empirical investigation is the correlation analysis between indices using Pearson's r . The purpose of this method is to further elucidate similarities and differences among indices. As is expected, based on the results presented in the correlation matrix in Table 7, there is very strong correlation among the summary between-seasons indices verifying that those indices capture similar aspects of competitive balance. On the other hand, there are cases with either considerably weak or even insignificant correlation. More specifically, the correlation of DN^I with DN_1 (0.272) and ADN_K (0.338) indices is quite weak. This may be justified by the different qualities those indices possess: DN^I captures the mobility of teams at the bottom,

whereas the other two indices capture the mobility of teams at the top of the ladder. We may interpret this finding by arguing that hardly ever do the I relegated teams come from the top K positions. Alternatively, the ranking mobility of the top K teams, and especially of the champion, is virtually independent of the ranking mobility of the relegated ones. This is equivalent to the scarceness of cases when promoted teams become champions the following season. A similar interpretation may be drawn from the fact that the correlation of aG with the DN^I (0.179) and the DN_1 (0.175) indices was not found to be significant. The former signifies that ranking mobility at the top is practically independent of the mobility at the bottom of the ladder while the latter may be interpreted by the fact that the champion's mobility is independent of the mobility of the remaining top teams. It must be noted that the champion's mobility is much lower than that of the remaining top teams.

Table 7: Correlation Matrix of the Indices

	τ	r_s	DN_t^*	DN_1	DN^I	ADN_K	SDN_K^I	aG
τ	1	0.979**	0.954**	0.324*	0.567**	0.569**	0.654**	0.639**
r_s		1	0.953**	0.361**	0.569**	0.611**	0.690**	0.651**
DN_t^*			1	0.305*	0.538**	0.549**	0.632**	0.603**
DN_1				1	0.272*	0.744**	0.737**	0.175
DN^I					1	0.338*	0.526**	0.179
ADN_K						1	0.975**	0.563**
SDN_K^I							1	0.552**
aG								1

*Significant at $\alpha=10\%$, **Significant at $\alpha=5\%$.

5. Discussion and Conclusions

A systematic approach for an enhanced quantification of between-seasons competitive balance in European football is offered by the review of existing and the development of new indices. Two important issues emerge from the review as far as the proper application of some indices is concerned. In particular, due to the promotion-relegation rule, which greatly affects the identity of a league across seasons thus making it difficult to accurately calculate competitive balance, a number of indices are excluded from the analysis. Moreover, for a reliable calculation and similar

definition of competitive balance boundaries, a modification is attempted by means of normalisation or re-location. The championship structure of European football leagues is argued to be even more complex given that the top teams qualify to participate in European tournaments the following season (Manasis et al. 2013). Therefore, the development of specially designed indices is attempted using the systematic approach offered by Manasis et al. (2013) for the seasonal dimension. A set new indices that weight ranking positions according to their importance and capture the multi-levelled structure of European football is introduced. The application of the new indices offers a powerful tool for an in-depth analysis of competitive balance by revealing interesting facts for league officials.

The empirical investigation, which employs data from the Premier League for the last 53 seasons, further elucidates the key features of all discussed indices. Based on the empirical results, the fact that the value of the between-seasons competitive balance in Premier League is closer to complete imbalance is of particular concern. As an interpretation, regardless of the uncertainty during the season, the stronger team finally prevails. In particular, competition for the championship title reaches the highest values which are very close to complete imbalance. This is indicative of less competition for the first as compared to the remaining ranking positions, which may be interpreted as the champion's negative contribution to a balanced league. This confirms the findings of Manasis et al. (2013) for the effectiveness of the promotion-relegation rule in promoting seasonal competitive balance in England. Therefore, if we ignore this mechanism, competitive balance is considerably inferior than it appears. From the correlation analysis may be derived that it is rarely a case that one of the top K teams is relegated or one of the promoted teams becomes the champion in the following season. The interpretation may be that the ranking in the previous season determines the success for the championship title rather than relegation. Alternatively, a large number of teams are candidates for relegation in contrast to a small number of teams that are candidates for the championship title.

Our suggestion is to thoroughly examine all indices based on the *Uncertainty of Outcome Hypothesis (UOH)* (Fort and Maxcy 2003; Zimbalist 2002; Zimbalist 2003). A proper econometric study is likely to reveal which indices, mostly affect the demand for football games or for other associated league products. Therefore, we

assume that both the significance and the effect on the demand for football products will determine the usefulness of the each index from the fans' perspective.

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