Assessing the Performance of a Prediction Error Criterion Model Selection Algorithm

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Abstract

Autoregressive Conditional Heteroscedasticity (ARCH) models have successfully been applied in order to predict asset return volatility. Predicting volatility is of great importance in pricing financial derivatives, selecting portfolios, measuring and managing investment risk more accurately. In this paper, a number of ARCH models are examined in the framework of a method for model selection based on a prediction error criterion (PEC) and their ability to predict future volatility is examined. According to this method, the ARCH model with the lowest sum of squared standardized forecasting errors is selected for predicting future volatility. A number of evaluation criteria are used to examine the performance of a model to predict future volatility, for forecasting horizons ranging from one day to one hundred days ahead. The results show that the PEC model selection procedure has a satisfactory performance in selecting that model that generates "better" volatility predictions. It appears, therefore, that it can be regarded as a tool in guiding one's choice of the appropriate model for predicting future volatility, with applications in evaluating portfolios, managing financial risk and creating speculative strategies with options.

Keywords and Phrases: ARCH models, Forecast Volatility, Model selection, Predictability, Correlated Gamma Ratio Distribution, Prediction Error Criterion

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Introduction

To evaluate their accuracy, volatility forecasts have to be compared with realized volatility, which cannot be observed. In the literature, it is common practice to refer the observed squared returns as the actual volatility. In this paper, a number of evaluation criteria are used to examine the ability of the PEC model selection algorithm introduced by Degiannakis and Xekalaki (1999, 2001) to indicate the ARCH model that generates "better" volatility predictions, for a forecasting horizon ranging from one day to one hundred days ahead. In the sequel, it is shown that the PEC algorithm has a satisfactory performance in selecting that ARCH model that tracks realized volatility closer, for a forecasting horizon ranging from 16 days to 36 days ahead. So, it is possible to use this model selection method in financial applications requiring volatility forecasts for a period longer than one day, i.e. option pricing, risk management. In section 2 of the paper, the ARCH process is presented. Section 3 describes the PEC model selection algorithm in the context of ARCH models. Section 4 provides a brief description of the evaluation criteria and the realized volatility measures considered. In section 5, the ability of the method proposed to select the ARCH model that generates "better" predictions of the volatility, is examined. In section 6, the proposed model selection method is compared to other methods of model selection Finally, in section 7, the construction of an alternative volatility measure based on intra-day data is discussed and, in section 8, a brief discussion of the results is provided.

1. *The Autoregressive Conditional Heteroscedasticity (ARCH) Process*

For P_t denoting the price of an asset at time t, let $y_t = \ln(P_t / P_{t-1})$ denote the continuously compounded return series of interest. The return series is decomposed into two parts, the predictable and unpredictable component:

$$y_t = E(y_{t|t-1}) + \varepsilon_t \, , \tag{2.1}$$

where $E(y_{t|t-1})$ is the conditional mean of return at period t depending upon the information set available at time $t-1$ and ε_t is the prediction error. Usually, the predictable component is either the overall mean or a first order autocorrelated process (imposed by non-synchronous trading[1]). The conditional mean, unfortunately, does not have the ability to give useful predictions. That is why modern financial theory assumes the asset returns are unpredictable. Before the start of the 1980's, the view taken about returns in financial markets was that they behave as random walks and the Brock et al. (1987) (BDS) statistic has widely been used to test the null hypothesis that asset returns are independently and identically distributed. This hypothesis, however, has been rejected in a vast number of applications. A rejection of the null hypothesis is consistent with some types of dependence in the data, which could result in from a linear stochastic system, a nonlinear stochastic system, or a nonlinear deterministic system. Thus, a question arises: "Are the nonlinearities connected with the conditional mean (so, as to be used to predict future returns) or with higher order conditional moments?" Artificial neural networks[2], chaotic dynamical systems[3], nonlinear parametric and nonparametric models[4] are some examples from the literature dealing with conditional mean predictions. ARCH models[5] and Stochastic Volatility models[6] are examples from the literature dealing with conditional variance modeling. However, no nonlinear models that can significantly outperform even the simplest linear model in out-of-sample forecasting seem to exist in the literature (neither in the field of stochastic nonlinear models nor in the field of deterministic chaotic systems). On the other hand, the ARCH processes and Stochastic Volatility models appear to be more appropriate to interpret nonlinearities in financial systems on the basis of the conditional variance. If an ARCH process is the true data generating mechanism, the nonlinearities cannot be exploited to generate improved point predictions relative to a linear model.

In the sequel, the conditional mean is considered as an κ^{th} order autoregressive process defined by

$$E\left(y_{t|t-1}\right) = c_0 + \sum_{i=1}^{\kappa} c_i y_{t-i} \ . \tag{2.2}$$

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[1] According to Campbell et al. (1997), "The non-synchronous trading or non-trading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular lengths." For more details on non-synchronous trading see Scholes and Williams (1977), Dimson (1979), Cohen et al. (1983), Lo and MacKinlay (1988, 1990), Campbell et al. (1997).

[2] For an overview of the Neural Networks literature see Poggio and Girosi (1990), Hertz et al. (1991), White (1992), Hutchinson et al. (1994).

[3] Brock (1986), Holden (1986), Thompson and Stewart (1986) and Hsieh (1991) review applications of chaotic systems to financial markets.

[4] Priestley (1988), Tong (1990) and Teräsvirta et al. (1994) cover a wide variety of nonlinear models.

[5] For an overview of the ARCH literature see Bollerslev et al. (1992), Bollerslev et al. (1994), Bera and Higgins (1993), Hamilton (1994), Gourieroux (1997).

[6] See for details Taylor (1994) and Shephard (1995).

Assuming the unpredictable component in (2.1) is an ARCH process, it can be represented as:

$$\begin{gathered} \varepsilon_t = z_t \sigma_t \\ z_t \overset{iid}{\sim} N(0,1) \\ \sigma_t^2 = g(\sigma_{t-1}(\theta), \sigma_{t-2}(\theta), \ldots; \varepsilon_{t-1}(\theta), \varepsilon_{t-2}(\theta), \ldots; \upsilon_{t-1}, \upsilon_{t-2}, \ldots), \end{gathered} \tag{2.3}$$

where $\{z_t\}$ is a sequence of independently and identically distributed random variables, $\mathrm{E}(z_t) = 0$, $\mathrm{V}(z_t) = 1$ and σ_t is a time-varying, positive measurable function of the information set at time $t-1$, υ_t is a vector of predetermined variables included in I_t and $g(.)$ could be a linear or nonlinear functional form as is usually assumed in the ARCH literature. The unpredictable component has variance σ_t^2, conditional on information given at time $t-1$. The conditional variance is a linear or nonlinear function of lagged conditional variances, past prediction errors and predetermined variables measurable at time $t-1$. The conditional prediction error is normally distributed, but the unconditional prediction error and the conditional variance of it have an unknown form of distribution. The conditional standardized prediction error, $z_{t|t-1}$, is standard normally distributed:

$$\varepsilon_{t|t-1} \sim N(0, \sigma_t^2) \Leftrightarrow z_{t|t-1} \equiv \varepsilon_{t|t-1} \sigma_t^{-1} \sim N(0,1). \tag{2.4}$$

In the recent literature, one can find a vast number of parametric specifications of ARCH models motivated by the characteristics explored in financial markets. A researcher, who is looking for the "best" model, would have in mind a variety of candidate models. The most commonly used conditional variance functions are the GARCH (Bollerslev (1986)), the Exponential GARCH, or E-GARCH, (Nelson (1991)) and the Threshold GARCH, or TARCH, (Glosten et al. (1993)) functions. In the sequel, these ARCH models are considered in the following forms:

The GARCH(p,q) model

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} \left(a_i \varepsilon_{t-i}^2\right) + \sum_{i=1}^{p} \left(b_i \sigma_{t-i}^2\right) \tag{2.5}$$

The E-GARCH(p,q) model

$$\ln(\sigma_t^2) = a_0 + \sum_{i=1}^{q} \left(a_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) \right) + \sum_{i=1}^{p} \left(b_i \ln(\sigma_{t-i}^2) \right) \tag{2.6}$$

The TARCH(p,q) model

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} \left(a_i \varepsilon_{t-i}^2\right) + \gamma \varepsilon_{t-1}^2 d_{t-1} + \sum_{i=1}^{p} \left(b_i \sigma_{t-i}^2\right), \tag{2.7}$$

where $d_t = 1$ if $\varepsilon_t < 0$, and $d_t = 0$ otherwise.

Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm (Marquardt (1963)), a modification of the Berndt, Hall, Hall and Hausman, or BHHH, algorithm (Berndt et al. (1974)). The quasi-maximum likelihood estimator (QMLE) is used, as according to Bollerslev and Wooldridge (1992), it is generally consistent, has a normal limiting distribution and provides asymptotic standard errors that are valid under non-normality.

The majority of practical applications, i.e. option pricing, determination of the value-at-risk, require more than one-day-ahead volatility forecasts. More than one-step-ahead forecasts can be computed by repeated substitution. The forecast recursion relation of the GARCH(p,q) model is:

$$\hat{\sigma}^2_{t+1|t} = a_0^{(t)} + \sum_{i=1}^{q}\left(a_i^{(t)}\varepsilon^2_{t-i+1}\right) + \sum_{i=1}^{p}\left(b_i^{(t)}\sigma^2_{t-i+1}\right) \tag{2.8.a}$$

$$\hat{\sigma}^2_{t+s|t} = a_0^{(t)} + \sum_{\substack{i=1 \\ for\ i<s}}^{q}\left(a_i^{(t)}\sigma^2_{t-i+s}\right) + \sum_{\substack{i=s \\ for\ i\geq s}}^{q}\left(a_i^{(t)}\varepsilon^2_{t-i+s}\right) + \sum_{i=1}^{p}\left(b_i^{(t)}\sigma^2_{t-i+s}\right), \tag{2.8.b}$$

For $s > t$, the forecast of predictive error ε_s conditional on information available at time t equals to its zero expected value, $E(\varepsilon_s \mid I_t) = 0$. On the other hand, the estimated value of ε_s^2 measured at time t should be equal to $\sigma^2_{s|t}$ for $s > t$. For $s \leq t$, the predictive error and its square are computed by the model with the available information at time t. The forecast recursion relationship associated with the E-GARCH(p,q) model is:

$$\ln\left(\hat{\sigma}^2_{t+1|t}\right) = a_0^{(t)} + \sum_{i=1}^{q}\left(a_i^{(t)}\left|\frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}}\right| + \gamma_i^{(t)}\left(\frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}}\right)\right) + \sum_{i=1}^{p}\left(b_i^{(t)}\ln\left(\sigma^2_{t-i+1}\right)\right) \tag{2.9.a}$$

$$\ln\left(\hat{\sigma}^2_{t+s|t}\right) = a_0^{(t)} + \sum_{\substack{i=s \\ for\ i\geq s}}^{q}\left(a_i^{(t)}\left|\frac{\varepsilon_{t-i+s}}{\sigma_{t-i+s}}\right| + \gamma_i^{(t)}\left(\frac{\varepsilon_{t-i+s}}{\sigma_{t-i+s}}\right)\right) + \sqrt{\frac{2}{\pi}}\sum_{\substack{i=1 \\ for\ i<s}}^{q}\left(a_i^{(t)}\right) + \sum_{i=1}^{p}\left(b_i^{(t)}\ln\left(\sigma^2_{t-i+s}\right)\right), \tag{2.9.b}$$

that associated with the TARCH(p,q) model is:

$$\hat{\sigma}^2_{t+1|t} = a_0^{(t)} + \sum_{i=1}^{q}\left(a_i^{(t)}\varepsilon^2_{t-i+1}\right) + \gamma^{(t)}\varepsilon_t^2 d_t + \sum_{i=1}^{p}\left(b_i^{(t)}\sigma^2_{t-i+1}\right) \tag{2.10.a}$$

$$\hat{\sigma}^2_{t+s|t} = a_0^{(t)} + \sum_{\substack{i=1 \\ for\ i<s}}^{q}\left(a_i^{(t)}\sigma^2_{t-i+s}\right) + \sum_{\substack{i=1 \\ for\ i\geq s}}^{q}\left(a_i^{(t)}\varepsilon^2_{t-i+s}\right) + \gamma^{(t)}\sigma^2_{t-1+s}E(d_t) + \sum_{i=1}^{p}\left(b_i^{(t)}\sigma^2_{t-i+s}\right). \tag{2.10.b}$$

Here, $E(d_t)$ denotes the percentage of negative innovations out of all innovations. Under the assumption of normally distributed innovations, the expected number of negative shocks is equal to the expected number of positive shocks, or $E(d_t) = 0.5$.

The forecast of the conditional variance at time t over a horizon of N days ahead is simply the average of the estimated future variance conditional on information given at time t is given by

$$\sigma^2_{t(N)} = N^{-1} \sum_{i=1}^{N} \hat{\sigma}^2_{t+i|t} \ . \tag{2.11}$$

2. *The Prediction Error Criterion (PEC) as a Model Selection Method*

Degiannakis and Xekalaki (1999) compare the forecasting ability of ARCH models using the Correlated Gamma Ratio (CGR) distribution. This is a distribution derived by Panaretos et al. (1997) as the distribution of the ratio of two variables jointly distributed according to Kibble's (1941) Bivariate Gamma distribution. Kibble (1941) proves that if, for $t = 1,2,...$, the joint distribution of $\left(r_t^{(A)}, r_t^{(B)}\right)$ is the Bivariate Standard Normal, then the joint distribution of $T^{-1}\sum_{t=1}^{T} r_t^{(A)2}$ and $T^{-1}\sum_{t=1}^{T} r_t^{(B)2}$ is Kibble's Bivariate Gamma distribution. As pointed out by Panaretos et al. (1997), $r_t^{(A)}$ and $r_t^{(B)}$ could represent the standardized prediction errors from two regression models (not necessarily nested) but with a common dependent variable. The distribution of the ratio of the sum of their squares is the Correlated Gamma Ratio distribution, $\sum_{t=1}^{T} r_t^{(B)2} \Big/ \sum_{t=1}^{T} r_t^{(A)2} \sim CGR(k,\rho)$, where $k = T/2$ and $\rho = Cor\left(r_t^{(A)}, r_t^{(B)}\right)$. Thus, two regression models can be compared through testing the null hypothesis of equivalence of the models in their predictability against the alternative that model (A) produces "better" predictions. The null hypothesis is rejected if $\sum_{t=1}^{T} r_t^{(B)2} \Big/ \sum_{t=1}^{T} r_t^{(A)2} > CGR(k,\rho,\alpha)$, where $CGR(k,\rho,a)$ is the $100(1-\alpha)$ percentile of the CGR distribution. (Percentage points of the CGR distribution can be found in Panaretos et al. (1997).

Let us now assume that we are interested in comparing the predictive ability of two ARCH models:

Model A	Model B
$\varepsilon_t^{(A)} = z_{1,t}\sigma_t^{(A)}$ $z_{1,t} \overset{iid}{\sim} N(0,1)$ $\sigma_t^{2(A)} = g\left(\sigma_{t-1}^{2(A)},...,\sigma_{t-p}^{2(A)},\varepsilon_{t-1}^{2(A)},...,\varepsilon_{t-q}^{2(A)},\upsilon_{t-1}^{(A)},\upsilon_{t-2}^{(A)},...\right)$	$\varepsilon_t^{(B)} = z_{2,t}\sigma_t^{(B)}$ $z_{2,t} \overset{iid}{\sim} N(0,1)$ $\sigma_t^{2(B)} = g\left(\sigma_{t-1}^{2(B)},...,\sigma_{t-p}^{2(B)},\varepsilon_{t-1}^{2(B)},...,\varepsilon_{t-q}^{2(B)},\upsilon_{t-1}^{(B)},\upsilon_{t-2}^{(B)},...\right)$

The joint distribution of $T^{-1}\sum_{t=1}^{T}\hat{z}_{t|t-1}^{2(A)} \equiv T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_{t|t-1}^{2(A)} \big/ \hat{\sigma}_{t|t-1}^{2(A)}$ and $T^{-1}\sum_{t=1}^{T}\hat{z}_{t|t-1}^{2(B)} \equiv T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_{t|t-1}^{2(B)} \big/ \hat{\sigma}_{t|t-1}^{2(B)}$ is Kibble's Bivariate Gamma distribution. Thus, the standardized one-step-ahead prediction errors can be used to test the null hypothesis of equivalence of the models in their predictive ability against the alternative that the first model produces "better" predictions. The null hypothesis is rejected if $\sum_{t=1}^{T}\hat{z}_{t|t-1}^{2(B)} \Big/ \sum_{t=1}^{T}\hat{z}_{t|t-1}^{2(A)} > CGR(k,\rho,\alpha)$.

According to the PEC model selection algorithm, the models that are considered as having a "better" ability to predict future values of the dependent variable, are those with the lowest sum of squared standardized one-step-ahead prediction errors. It becomes evident, therefore, that these models can potentially be regarded as the most appropriate to use for volatility forecasts too.

Let us assume that M candidate ARCH models are available and that we are looking for the "most suitable" model at each of a sequence of points in time. At time k, selecting a strategy for the most appropriate model to forecast volatility at time $k+1$ ($k = T, T+1,...$) could naturally amount to selecting the model which, at time k, has the lowest sum of squared standardized one-step-ahead prediction errors, on the basis of the PEC algorithm. Table 3.1 summarizes the estimation steps comprising this approach.

In the next section, the methodology applied to evaluate the performance of a model in estimating future volatility is presented, while in section 5, the ability of the PEC model selection algorithm to indicate those ARCH models that generate "better" volatility predictions is illustrated on a set of real data on daily returns of the S&P500 stock index.

3. *Evaluating the Volatility Forecast Performance*

The main problem in evaluating the predictive performance of a model is the choice of the function one should use to measure the distance between estimations and observations. Evaluating the performance of the variance forecasts requires knowledge of the actual volatility, which is unobservable. Thus, in evaluating the predictive performance of a variance model a question of a dual nature arises: that of determining the realized volatility and of considering the appropriate measure to evaluate the closeness of the forecasts to the corresponding realizations.

Table (3.1)

The estimation steps required at time k for each model m by the PEC model selection algorithm. (At time k ($k = T, T+1, \ldots$), select the model m with minimum $\sum_{t=k-T+1}^{k} \hat{z}_{t|t-1}^{2(m)}$.)

Model	Time: $k = T$	$k = T+1$	. . .	$k = T+K$	. . .
$m = 1$	$\sum_{t=1}^{T} \hat{z}_{t\|t-1}^{2(1)}$	$\sum_{t=2}^{T+1} \hat{z}_{t\|t-1}^{2(1)}$	. . .	$\sum_{t=K+1}^{T+K} \hat{z}_{t\|t-1}^{2(1)}$	. . .
$m = 2$	$\sum_{t=1}^{T} \hat{z}_{t\|t-1}^{2(2)}$	$\sum_{t=2}^{T+1} \hat{z}_{t\|t-1}^{2(2)}$	. . .	$\sum_{t=K+1}^{T+K} \hat{z}_{t\|t-1}^{2(2)}$	. . .
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	
$m = M$	$\sum_{t=1}^{T} \hat{z}_{t\|t-1}^{2(M)}$	$\sum_{t=2}^{T+1} \hat{z}_{t\|t-1}^{2(M)}$	. . .	$\sum_{t=K+1}^{T+K} \hat{z}_{t\|t-1}^{2(M)}$	. . .

4.1 Realized Volatility Measures

Practitioners' most popular volatility measures are the average of squared daily returns and the variance of the daily returns. These measures, expressed on a daily basis for a horizon of N days ahead, are:

$$\breve{s}_{t(N)}^{2} = N^{-1} \sum_{i=1}^{N} y_{t+i}^{2} , \quad (4.1)$$

$$\hat{s}_{t(N)}^{2} = (N-1)^{-1} \sum_{i=1}^{N} \left(y_{t+i} - \overline{y}_{t(N)} \right)^{2} , \quad (4.2)$$

respectively, where $\bar{y}_{t(N)} = N^{-1}\sum_{i=1}^{N} y_{t+i}$ is the average return.

4.2 Evaluation Criteria

A large number of forecast evaluation criteria exist in the literature. However, none is generally acceptable. Because of high non-linearity in volatility models and the variety of statistical evaluation criteria, a number of researchers constructed economic criteria based upon the goals of their particular application. West et al. (1993) develop a criterion based on the decisions of a risk averse investor. Engle et al. (1993) assume that the objective is to price options and develop a loss function from the profitability of a particular trading strategy. In the sequel, we focus on statistical criteria to measure the closeness of the forecasts to the realizations, in order to avoid restrictions imposed by economic theory. Moreover, we consider statistical criteria that are robust to non-linearity and heteroscedasticity. Pagan and Schwert (1990) use statistical criteria to compare parametric and non-parametric ARCH models with in-sample and out-of-sample data. Besides, Heynen and Kat (1994) investigate the predictive performance of ARCH and Stochastic Volatility models and Hol and Koopman (2000) compare the predictive ability of Stochastic Volatility and Implied Volatility models. Andersen et al. (1999b) applied heteroscedasticity-adjusted statistics to examine the forecasting performance of intraday returns. Denoting the forecasting variance over an N day period measured at day t by $\sigma^2_{t(N)}$, and the realized variance over the same period by $s^2_{t(N)}$, the following evaluation criteria are considered:

Squared Error (SE): $\left(\sigma^2_{t(N)} - s^2_{t(N)}\right)^2$ (4.3)

Absolute Error (AE): $\left|\sigma^2_{t(N)} - s^2_{t(N)}\right|$ (4.4)

Heteroscedasticity Adjusted Squared Error (HASE): $\left(1 - s^2_{t(N)}\big/\sigma^2_{t(N)}\right)^2$ (4.5)

Heteroscedasticity Adjusted Absolute Error (HAAE): $\left|1 - s^2_{t(N)}\big/\sigma^2_{t(N)}\right|$ (4.6)

Logarithmic Error (LE): $\ln\left(s^2_{t(N)}\big/\sigma^2_{t(N)}\right)^2$ (4.7)

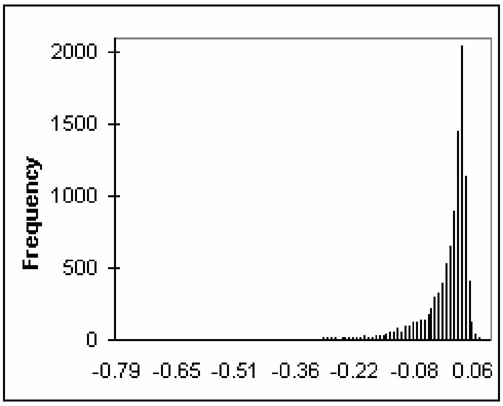
The first two functions have been widely used in the literature (see, e.g. Heynen and Kat (1994), West and Cho (1995)). The HASE and HAAE functions were considered by Andersen et al. (1999b), while the LE function was utilized by Pagan and Schwert (1990).

Usually, the average of the evaluation criteria is considered. However, when simulating an AR(1)GARCH(1,1) process, which is the most commonly used model in financial applications, the distributions of $\left(\sigma^2_{t(N)} - s^2_{t(N)}\right)$, $\left(1 - s^2_{t(N)}/\sigma^2_{t(N)}\right)$ and $\ln\left(s^2_{t(N)}/\sigma^2_{t(N)}\right)$ are asymmetric with extreme outliers. It would therefore be advisable to compute both the mean and the median of the evaluation criteria. Figure 4.1 depicts the histograms of the one-step forecast error distribution from the following simulated process:

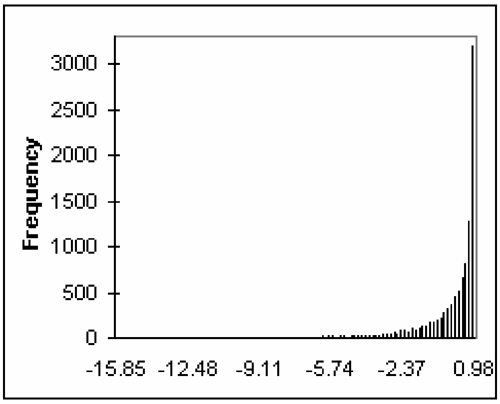
$$\begin{aligned} y_t &= 0.001 + 0.1 y_{t-1} + \varepsilon_t \\ \sigma^2_t &= 0.002 + 0.05\varepsilon^2_{t-1} + 0.9\sigma^2_{t-1} \\ \varepsilon_t &= \sigma_t z_t \quad \text{and} \quad z_t \overset{iid}{\sim} N(0,1). \end{aligned} \tag{4.8}$$

Figure 4.1.

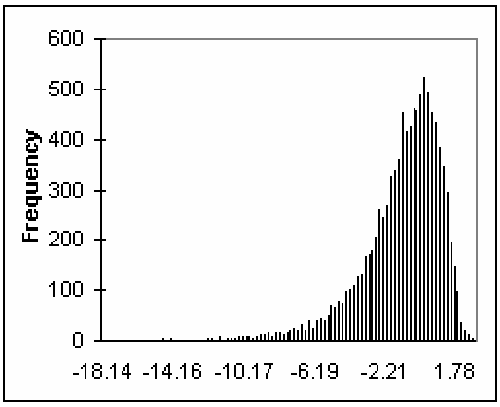
Histogram of $\sigma^2_{t(1)} - y^2_t$ from an AR(1)GARCH(1,1) simulated process



Histogram of $1 - y^2_t/\sigma^2_{t(1)}$ from an AR(1)GARCH(1,1) simulated process



Histogram of $\ln\left(y^2_t/\sigma^2_{t(1)}\right)$ from an AR(1)GARCH(1,1) simulated process



4. *Examining the Performance of the PEC Model Selection Algorithm*

In this section, the ability of the PEC model selection algorithm to lead to the ARCH models that track closer future volatility is illustrated on a real stock index daily return series. As follows from section 2, the return series can be modeled in the following form:

$$\begin{aligned} y_t &= E\left(y_{t|t-1}\right) + \varepsilon_t \\ E\left(y_{t|t-1}\right) &= c_0 + \sum_{i=1}^{\kappa} c_i y_{t-i} \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\overset{iid}{\sim} N(0,1) \\ \sigma_t^2 &= g\left(\sigma_{t-1}(\theta), \sigma_{t-2}(\theta), \ldots; \varepsilon_{t-1}(\theta), \varepsilon_{t-2}(\theta), \ldots; \upsilon_{t-1}, \upsilon_{t-2}, \ldots\right) \end{aligned} \qquad (5.1)$$

In the sequel, the above form is considered in connection with the ARCH models defined by (2.5), (2.6) and (2.7), for $\kappa = 0,1,2,3,4$, $p = 0,1,2$ and $q = 1,2$, thus yielding a total of 85 cases[7].

The data set consists of 1661 S&P500 stock index daily returns in the period from November 24th, 1993 to June 26th, 2000. The ARCH processes are estimated using a rolling sample of constant size equal to 500. Thus, the first one-step-ahead volatility prediction, $\hat{\sigma}^2_{t+1|t}$, is available at time $t = 500$. Applying the PEC model selection algorithm, the sum of squared standardized one-step-ahead prediction errors, $\sum_{t=1}^{T} \hat{z}^2_{t|t-1}$, was estimated considering various values for T, and, in particular, $T = 5(5)80$. This is an indirect way to examine the performance of the PEC model selection algorithm for various values of T. Thus, the evaluation criteria were applied on the one-step-ahead forecasts using $1661 - 500 - 80 = 1081$ data points, on the two-step-ahead forecasts using $1661 - 500 - 81 = 1080$ data points, …, and on the kth-step-ahead forecasts using $1081 - k + 1$ data points.

Our main purpose is to examine the application potential of the PEC algorithm of selection of models on the basis of their forecasting ability in terms of volatility. So, the mean and the median value of each of the 5 evaluation criteria, in equations (4.3)-(4.7),

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[7] Numerical maximization of the log-likelihood function, for the E-GARCH(2,2) model, frequently failed to converge. So the five E-GARCH models for $p = q = 2$ were excluded.

were computed, yielding a total of 10 evaluation criteria for each forecasting horizon from one day to one hundred days ahead. However, volatility is expressed either as the variance or as the standard deviation. Thus, in order to examine possible differences between forecasting the variance and its square root, the evaluation criteria were, also, applied on the standard deviation. Therefore, $\sigma^2_{t(N)}$ and $s^2_{t(N)}$, in equations (4.3)-(4.7), were replaced by $\sigma_{t(N)}$ and $s_{t(N)}$, respectively and 10 more evaluation criteria were computed. In total, 20 evaluation criteria were computed for a horizon ranging from one day to five months. In section 4.1, two realized volatility measures were mentioned. The results are based on the realized volatility as defined by $\breve{s}^2_{t(N)}$. (Results based on the realized volatility as defined by $\widehat{s}^2_{t(N)}$ are similar and are not reported.)

It was examined whether the ARCH models selected by the PEC algorithm achieve the lowest value of the evaluation criteria. The main focus was on the median values of the criteria and mainly on the heteroscedasticity adjusted criteria since they are more robust to asymmetry. Table A.1, in the Appendix, presents the minimum and maximum values of the evaluation criteria that were achieved by each of the 85 ARCH models and the ARCH models suggested by the PEC model selection algorithm. The PEC algorithm is applied for 16 values for T, and, in particular, $T = 5(5)80$. The minPEC (maxPEC) value refers to the minimum (maximum) of the 16 values of the evaluation criteria achieved by the models selected by the PEC algorithm. Moreover, for each of the 85 estimated ARCH models the evaluation criteria have been computed. The minARCH (maxARCH) value refers to the minimum (maximum) of the 85 values of the evaluation criteria achieved by the ARCH models.

Figure A.1, in the Appendix, shows, for each evaluation criterion and each forecasting horizon, whether ARCH models selected by the PEC algorithm achieve the lowest value of the evaluation criteria. In the first part of Figure A.1, the performance of the models, which are selected by the PEC algorithm, on the basis of the conditional variance is depicted, while, the second part refers to their performance on forecasting standard deviation. The general conclusion is that the PEC algorithm led to the selection of the ARCH processes which track closer the realized volatility in the majority of the cases. Specifically, for the forecasting horizon ranging from 11 to 52 days, the models selected by the PEC algorithm achieve the lowest criteria values, irrespectively of the evaluation criteria. The percentage of cases, that the models selected by the PEC algorithm achieve the lowest value of the evaluation criteria, is higher around the forecasting horizon ranging from 16 to 36 days ahead, or 4 to 7 trading weeks ahead.

The ability of the PEC algorithm to select the ARCH models that generate "better" predictions of the volatility, around a forecasting horizon of 4 to 7 weeks ahead, is indicative of its usage potential in applications exploiting volatility forecasts as, for example, in pricing derivatives, estimating the risk of a portfolio etc. Table A.2, in the Appendix, presents the percentage of cases the models selected by the PEC algorithm perform "better" as judged by the evaluation criteria, for 3 different horizon ranges. Note that, in terms of the MSE and MAE criteria, none of the models chosen by the PEC algorithm appears to perform better in any of the forecasting horizons considered. But, in terms of the median values of the criteria and the heteroscedasticity adjusted criteria, which are robust to asymmetry, the models selected by the PEC algorithm appear to have a better performance in all the forecasting horizons considered.

It is interesting to note that, via the evaluation criteria considered, the optimum sample size, T, for the PEC model selection algorithm can be determined. The PEC model selection algorithm has been applied for $T = 5(5)80$. In the sequel, the value of T for which the PEC selection method achieves the best performance according to the evaluation criteria used, is examined. Figure 5.1 shows a plot of the average T, suggested by the evaluation criteria, across the forecasting horizons. The bar charts of Figure 5.1 are a graphical representation of the number of evaluation criteria by which the performance of the models selected by the PEC algorithm were judged "best" (measured on the right hand side vertical axis).

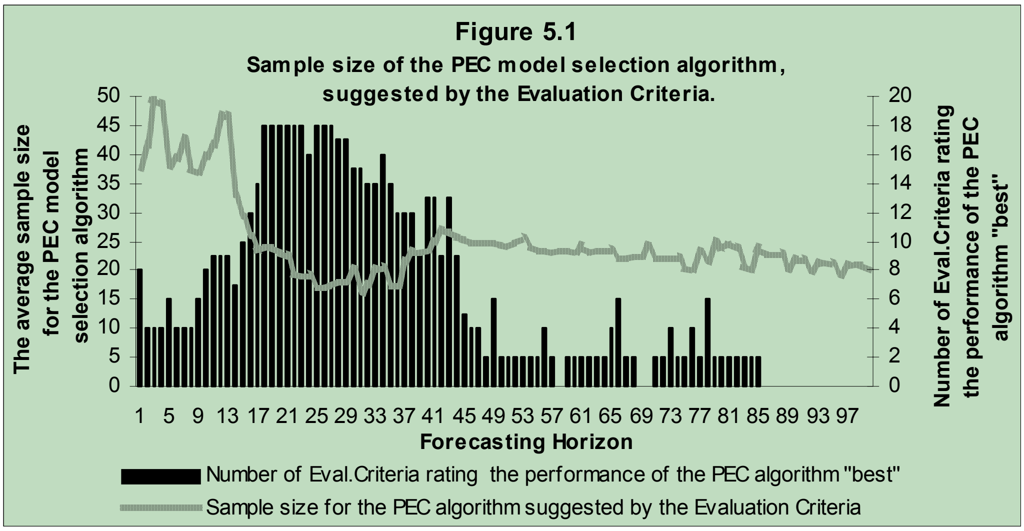
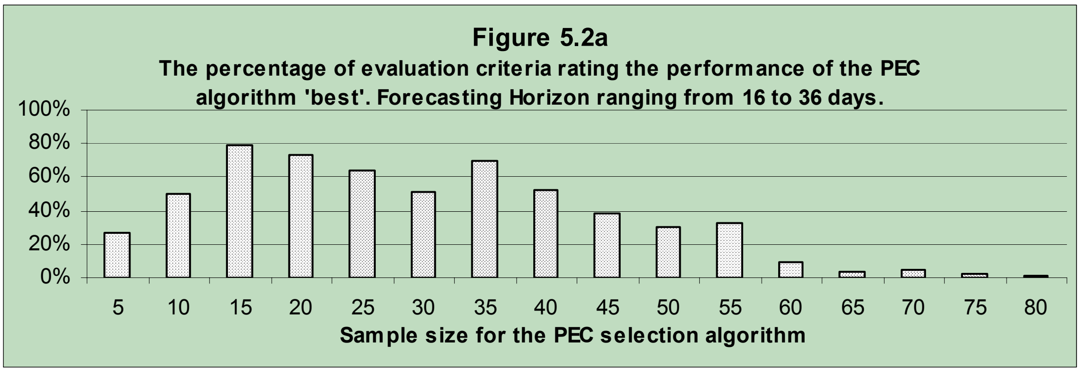
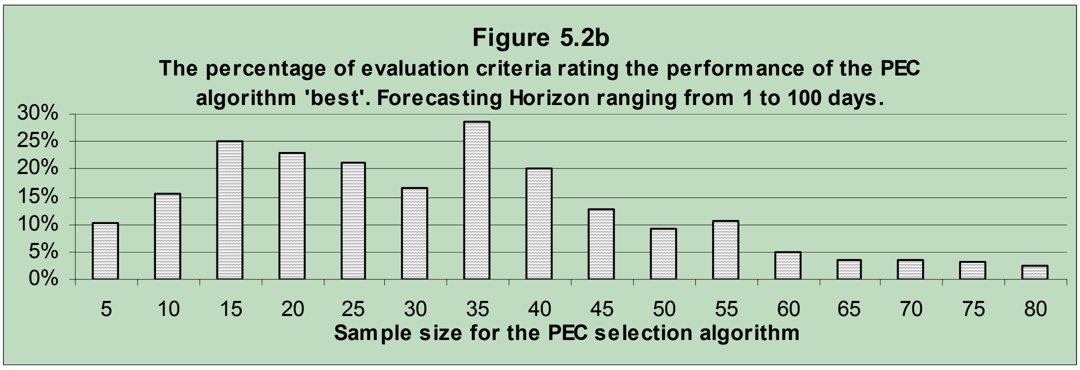


Figure 5.1
Sample size of the PEC model selection algorithm, suggested by the Evaluation Criteria.

For a 16 to 36 day ahead forecasting horizon, the appropriate T, as concerns the specific data, ranges around 20 days with a standard deviation of 3.6 days. Table A.3, in the Appendix, provides more details for the sample size of the PEC selection method suggested by the evaluation criteria and its standard deviation for both the entire 16 to 36 day ahead forecasting horizon and for each day individually. The PEC model selection algorithm shows a better performance for a sample size of about 20 days.

In order to test the importance of selecting the appropriate T, for the model selection method suggested, the evaluation criteria were run for $T = 5(5)80$. The results are indeed in support of a sample size of around 20 days for the PEC algorithm to manifest a better performance. Figure 5.2 presents the percentage of the evaluation criteria by which the PEC algorithm, with specific T, selects those ARCH models that generate "better" volatility predictions. For T ranging from 15 to 35, the PEC selection method appears to have the highest performance.





5. *Comparison of the PEC Criterion to Other Methods of Model Selection*

Most of the methods used in the time series literature for selecting the appropriate model are based on evaluating the ability of the models to describe the data. Standard model selection criteria such as the Akaike Information Criterion [AIC] (Akaike (1973)) and the Schwarz Bayesian Criterion [SBC] (Schwarz (1978)) have widely been used in the ARCH literature, despite the fact that their statistical properties in the ARCH context are unknown. These are defined in terms of $l_n(\hat{\theta})$, the maximized value of the log-likelihood function of a model, where $\hat{\theta}$ is the maximum likelihood estimator of the parameter vector θ based on a sample of size n and $\breve{\theta}$ denotes the dimension of θ, thus:

$$AIC = l_n(\hat{\theta}) - \breve{\theta} \quad (6.1)$$

$$SBC = l_n(\hat{\theta}) - 2^{-1}\breve{\theta}\ln(n). \quad (6.2)$$

In addition, model selection is mainly based on the evaluation of some loss function for each of the competing models. In this section, the statistical criteria, which were considered in section 4 as measures in evaluating the predictive performance of a variance model, are considered as criteria for the selection of ARCH models. In particular, the model selection methods presented in Table 6.1, are considered and their ability to predict future volatility is investigated.

Applying the PEC model selection algorithm, the sum of squared standardized one-step-ahead prediction errors, $\sum_{t=1}^{T}\hat{\varepsilon}_{t|t-1}^{2}/\hat{\sigma}_{t|t-1}^{2}$, was estimated considering various values for T. Therefore, each of the model selection criteria, in Table 6.1, was computed considering various values for T, and, in particular, $T = 10(10)80$. The AIC and SBC criteria were computed based on the rolling sample of constant size equal to 500, or $n = 500$, that is used at each time to estimate the parameters of the models. Based on Table 3.1, selecting a strategy for each method of model selection naturally amounts to selecting the model, which, at time k, has the lowest value of the formula is indicated in Table 6.1.

Table A.4, in the Appendix, presents the percentage of cases the models selected by each model selection method perform “better” as judged by the evaluation criteria, for 3 different horizon ranges. As concerns the AIC and SBC selection methods, they do not achieve the lowest value of the evaluation criteria in almost all the cases, which is indicative of the inability of the in-sample model selection methods to suggest the models with superior volatility forecasting performance. The general conclusion is that the loss functions presented in Table 6.1 do not led to the selection of the ARCH processes which track closer the realized volatility. The HASEVar, HAAEVar and HASEDev criteria show a better performance, as they select the ARCH models with the lowest value of the evaluation criteria, around the forecasting horizon ranging from 16 to 36 days ahead. So, they might be used in selecting that model that generates “better” volatility predictions. In order to investigate whether the suggested model selection method or the loss functions indicate the ARCH models that track closer the realized volatility, the predictive ability of these loss functions must be compared to the volatility forecasting ability of the PEC criterion, and mainly for a forecasting horizon ranging from 16 days to 36 days ahead.

Table (6.1)
Methods of selection of ARCH models.

1. Square Error of Conditional Variance (SEVar):

$$\sum_{t=1}^{T}\left(\left(\sigma_{t(N)}^{2}-s_{t(N)}^{2}\right)^{2}\right) \quad (6.3)$$

2. Absolute Error of Conditional Variance (AEVar):

$$\sum_{t=1}^{T}\left(\left|\sigma_{t(N)}^{2}-s_{t(N)}^{2}\right|\right) \quad (6.4)$$

3. Square Error of Conditional Standard Deviation (SEDev):

$$\sum_{t=1}^{T}\left(\left(\sigma_{t(N)}-s_{t(N)}\right)^{2}\right) \quad (6.5)$$

4. Absolute Error of Conditional Standard Deviation (AEDev):

$$\sum_{t=1}^{T}\left(\left|\sigma_{t(N)}-s_{t(N)}\right|\right) \quad (6.6)$$

5. Heteroscedasticity Adjusted Squared Error of Cond. Variance (HASEVar):

$$\sum_{t=1}^{T}\left(\left(1-s_{t(N)}^{2}/\sigma_{t(N)}^{2}\right)^{2}\right) \quad (6.7)$$

6. Heteroscedasticity Adjusted Absolute Error of Cond. Variance (HAAEVar):

$$\sum_{t=1}^{T}\left(\left|1-s_{t(N)}^{2}/\sigma_{t(N)}^{2}\right|\right) \quad (6.8)$$

7. Heteroscedasticity Adjusted Squared Error of Cond. St. Deviation (HASEDev):

$$\sum_{t=1}^{T}\left(\left(1-s_{t(N)}/\sigma_{t(N)}\right)^{2}\right) \quad (6.9)$$

8. Heteroscedasticity Adjusted Absolute Error of Cond. St. Deviation (HAAEDev):

$$\sum_{t=1}^{T}\left(\left|1-s_{t(N)}/\sigma_{t(N)}\right|\right) \quad (6.10)$$

9. Logarithmic Error of Conditional Variance (LEVar):

$$\sum_{t=1}^{T}\left(\ln\left(s_{t(N)}^{2}/\sigma_{t(N)}^{2}\right)^{2}\right) \quad (6.11)$$

10. Akaike Information Criterion (AIC):

$$AIC=l_{n}\left(\hat{\theta}\right)-\breve{\theta} \quad (6.12)$$

11. Schwarz Bayesian Criterion (SBC):

$$SBC=l_{n}\left(\hat{\theta}\right)-2^{-1}\breve{\theta}\ln(n) \quad (6.13)$$

Of main interest is whether the ARCH models selected by the PEC algorithm yield values for the evaluation criteria that are lower than those corresponding to the

ARCH models selected by the model selection methods summarized in Table 6.1. Table A.5, in the Appendix, presents the percentage of times the ARCH models selected by the PEC algorithm achieve lower values for the corresponding evaluation criteria and the specific forecasting horizons than the models selected by the other model selection methods. As concerns forecasting horizons of 4 to 7 trading weeks ahead, which are of focal interest in the majority of practical applications (e.g. in determining the value-at-risk), the performance of the PEC algorithm is by far the best.

The PEC model selection algorithm performs “better” than the other methods of model selection in about 90% of the cases. This percentage is lower when the PEC algorithm is compared to the HASEVar, HAAEVar and HASEDev methods. Nevertheless, even in such cases, the opponent methods select the ARCH models that track closer future volatility much less frequently than the PEC algorithm. The percentage of times, an opponent to the PEC algorithm selects the most appropriate models in forecasting future volatility, is highest in the case of the HAAEVar method. However, only in the 23% of cases, the ARCH models selected by the HAAEVar method perform "better" than the models selected by the PEC criterion, for any of the 3 horizon ranges.

6. *Intra-Day Realized Volatility Measures*

In section 4.1 the most popular volatility measures were presented. However, as noted in the literature (e.g. Ebens (1999)), although the squared daily returns are unbiased volatility estimators, they are very noisy. Under the ARCH process, the squared return can be represented by $y_t^2 = z_t^2 \sigma_t^2$. It is therefore defined as the product of the true volatility times the square of a normally distributed process. An alternative volatility measure, introduced by Andersen and Bollerslev (1997a), is the integrated volatility that is computed as the summation of the squared finely sampled high frequency data[8]. For P_t denoting the price of an asset at day t, let the difference of the log-prices,

$$y_{(m),t} = \ln(P_t) - \ln(P_{t-1/m}), \text{ where } t = 1/m, 2/m, \ldots, \quad (7.1)$$

[8] For more information about integrated volatility and the use of intraday data see Andersen and Bollerslev (1997a), (1997b), (1998), Andersen et al. (1998, 1999a), Andersen et al. (1999b), Andersen et al. (2000a), Andersen et al. (2000b), Barndorff-Nielsen and Shephard (1998), Ebens (1999).

denote the discretely observed series of continuously compounded returns with m observations per day. The integrated volatility, that is central to the option pricing in Hull and White (1987), over an interval of length h is defined as:

$$y_{h,t}^{2} = \int_{0}^{h} s_{t-h+r}^{2} dr \,, \tag{7.2}$$

where s_t is the volatility of the instantaneous returns process, generated by the continuous time martingale, $d\ln(P_t) = s_t dW_t$, (W_t is the standard Wiener process). In the case of discrete time with a sample frequency of $h = 1/m$, $y_{(1/h),t}^{2}$ is an unbiased estimator of $y_{h,t}^{2}$. However, for daily volatility forecasts, or $(h = 1)$, the discretely sampled daily returns, for $(m = 1)$, constitute a noisy estimator. But, as noted by Ebens (1999) and Andersen and Bollerslev (1997a), the accuracy improves as the sampling frequency is increasing, $(m \to \infty)$. So, the approximation to the integrated volatility on a daily basis for a horizon of N days ahead is:

$$\tilde{s}_{t(N)}^{2} = N^{-1} \sum_{i=1}^{N} \sum_{j=1}^{m-1} \left(\ln\left(P_{(j+1/m),t-i}\right) - \ln\left(P_{(j/m),t-i}\right)\right)^{2} \,. \tag{7.3}$$

Investigating the performance of the PEC criterion in selecting that model that generates “better” volatility predictions using the realized volatility measure computed in (7.3) would be interesting.

7. *Conclusions*

A method for selecting an ARCH model among several competing models was suggested. It amounts to selecting the model with the lowest sum of squared standardized forecasting errors.

A number of evaluation criteria, for forecasting horizons ranging from one day to one hundred days ahead, were applied and it was found that the ARCH models, selected by the PEC model selection algorithm, generate “better” predictions of the volatility. Thus, the PEC selection method appears to be a useful tool in guiding one’s choice of the appropriate model for estimating future volatility, with applications in evaluating portfolios, derivatives and financial risk.

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Appendix

Table A.1

The table presents the minimum and maximum values of the evaluation criteria that were achieved by each of the ARCH models and the ARCH models suggested by the PEC model selection algorithm, respectively, for a subset of the forecasting horizon which ranges from 2 to 100 days. The first panel refers to the variance, (k=2), whereas the second panel accounts for the standard deviation, (k=1). The evaluation criteria are the annualized mean and median values of the following loss functions:

Squared Error (SE) : $\left(\sigma_{t(T)}^{k} - s_{t(T)}^{k}\right)^2$

Absolute Error (AE) : $\left|\sigma_{t(T)}^{k} - s_{t(T)}^{k}\right|$

Heteroscedasticity Adjusted Squared Error (HASE) : $\left(1 - s_{t(T)}^{k} / \sigma_{t(T)}^{k}\right)^2$

Heteroscedasticity Adjusted Absolute Error (HAAE): $\left|1 - s_{t(T)}^{k} / \sigma_{t(T)}^{k}\right|$

Logarithmic Error (LE): $\ln\left(s_{t(T)}^{k} / \sigma_{t(T)}^{k}\right)^2$

Evaluation Criteria for the Conditional Variance, k=2

	Forecast Horizon in days																					
	5	10	15	16	18	20	22	24	26	28	30	32	34	35	40	45	50	60	70	80	90	100
MeanSE																						
minPEC	**0,187**	**0,126**	**0,097**	**0,092**	**0,083**	**0,077**	**0,072**	**0,069**	**0,067**	**0,065**	**0,064**	**0,063**	**0,062**	**0,061**	**0,059**	**0,056**	**0,053**	**0,049**	**0,045**	**0,041**	**0,038**	**0,036**
maxPEC	0,208	0,140	0,107	0,102	0,094	0,089	0,084	0,082	0,079	0,077	0,075	0,073	0,072	0,072	0,068	0,065	0,061	0,056	0,050	0,046	0,043	0,041
minARCH	**0,177**	**0,116**	**0,087**	**0,083**	**0,076**	**0,070**	**0,066**	**0,063**	**0,061**	**0,058**	**0,057**	**0,055**	**0,054**	**0,054**	**0,050**	**0,047**	**0,044**	**0,040**	**0,037**	**0,034**	**0,031**	**0,030**
maxARCH	0,220	0,160	0,129	0,123	0,114	0,107	0,101	0,098	0,096	0,094	0,094	0,094	0,094	0,095	0,095	0,094	0,093	0,092	0,091	0,092	0,091	0,091
MeanAE																						
minPEC	**1,964**	**1,683**	**1,541**	**1,513**	**1,458**	**1,423**	**1,402**	**1,395**	**1,386**	**1,378**	**1,366**	**1,360**	**1,361**	**1,363**	**1,356**	**1,353**	**1,351**	**1,340**	**1,329**	**1,317**	**1,305**	**1,291**
maxPEC	2,176	1,869	1,708	1,674	1,605	1,550	1,532	1,527	1,521	1,509	1,498	1,490	1,493	1,492	1,472	1,465	1,464	1,448	1,431	1,415	1,388	1,371
minARCH	**1,844**	**1,588**	**1,492**	**1,471**	**1,432**	**1,409**	**1,395**	**1,386**	**1,374**	**1,359**	**1,350**	**1,337**	**1,335**	**1,331**	**1,314**	**1,292**	**1,283**	**1,263**	**1,241**	**1,233**	**1,214**	**1,201**
maxARCH	2,217	1,963	1,840	1,812	1,761	1,722	1,701	1,692	1,677	1,667	1,660	1,656	1,666	1,670	1,671	1,677	1,681	1,692	1,702	1,701	1,692	1,687
MedSE																						
minPEC	**0,0119**	**0,0077**	**0,0066**	**0,0064**	**0,0055**	**0,0052**	**0,0051**	**0,0050**	**0,0050**	**0,0048**	**0,0047**	**0,0047**	**0,0045**	**0,0043**	**0,0047**	**0,0044**	**0,0046**	**0,0041**	**0,0045**	**0,0050**	**0,0053**	**0,0056**
maxPEC	0,0162	0,0115	0,0090	0,0090	0,0074	0,0068	0,0066	0,0063	0,0060	0,0058	0,0055	0,0054	0,0053	0,0055	0,0053	0,0056	0,0058	0,0061	0,0059	0,0064	0,0070	0,0074
minARCH	**0,0094**	**0,0068**	**0,0061**	**0,0056**	**0,0059**	**0,0057**	**0,0055**	**0,0056**	**0,0052**	**0,0049**	**0,0048**	**0,0048**	**0,0045**	**0,0047**	**0,0044**	**0,0043**	**0,0047**	**0,0043**	**0,0042**	**0,0048**	**0,0048**	**0,0045**
maxARCH	0,0151	0,0119	0,0098	0,0096	0,0084	0,0082	0,0083	0,0085	0,0085	0,0082	0,0079	0,0079	0,0075	0,0078	0,0082	0,0082	0,0092	0,0106	0,0113	0,0118	0,0121	0,0132
MedAE																						
minPEC	**1,089**	**0,877**	**0,815**	**0,799**	**0,742**	**0,723**	**0,717**	**0,708**	**0,708**	**0,690**	**0,684**	**0,683**	**0,668**	**0,658**	**0,683**	**0,661**	**0,681**	**0,642**	**0,672**	**0,706**	**0,729**	**0,749**
maxPEC	1,274	1,073	0,947	0,951	0,859	0,827	0,810	0,797	0,776	0,758	0,740	0,738	0,728	0,745	0,726	0,751	0,759	0,779	0,766	0,801	0,835	0,860
minARCH	**0,970**	**0,824**	**0,784**	**0,751**	**0,770**	**0,755**	**0,740**	**0,748**	**0,722**	**0,703**	**0,696**	**0,690**	**0,674**	**0,682**	**0,666**	**0,653**	**0,689**	**0,657**	**0,650**	**0,694**	**0,690**	**0,671**
maxARCH	1,228	1,089	0,988	0,981	0,915	0,906	0,909	0,920	0,923	0,907	0,890	0,890	0,868	0,881	0,906	0,906	0,958	1,028	1,063	1,088	1,100	1,150
MeanHASE																						
minPEC	**208,9**	**146,0**	**111,0**	**104,7**	**93,2**	**85,1**	**78,9**	**75,2**	**73,3**	**71,3**	**70,3**	**69,7**	**68,5**	**68,2**	**65,9**	**63,4**	**61,7**	**62,4**	**61,3**	**59,1**	**57,3**	**57,5**
maxPEC	250,3	170,1	131,0	128,1	121,0	114,6	113,9	111,1	109,0	106,1	103,2	101,0	98,6	97,7	93,3	89,5	85,8	81,9	76,3	71,1	67,2	66,2
minARCH	**236,8**	**158,5**	**117,6**	**111,5**	**99,5**	**91,9**	**86,5**	**82,1**	**78,9**	**76,6**	**75,0**	**73,7**	**72,5**	**72,3**	**68,1**	**64,6**	**60,5**	**57,2**	**54,5**	**51,5**	**48,1**	**47,9**
maxARCH	414,9	279,8	232,7	226,3	214,7	203,8	196,3	189,0	183,6	178,7	174,5	170,9	167,5	166,1	159,1	153,1	147,9	141,3	136,3	132,2	128,7	126,8
MeanHAAE																						
minPEC	**64,52**	**56,37**	**52,23**	**51,22**	**49,53**	**48,15**	**47,53**	**47,37**	**47,38**	**47,33**	**47,15**	**47,32**	**47,51**	**47,68**	**47,99**	**48,49**	**49,09**	**50,71**	**51,74**	**52,64**	**53,31**	**54,09**
maxPEC	68,19	60,02	56,63	56,30	55,61	55,29	55,49	55,52	55,48	55,14	54,76	54,63	54,79	54,78	54,32	54,61	55,13	55,98	56,32	56,65	56,86	57,32
minARCH	**70,92**	**61,82**	**57,56**	**56,56**	**54,71**	**53,32**	**52,69**	**52,21**	**51,80**	**51,33**	**50,95**	**50,90**	**51,11**	**51,18**	**51,17**	**50,88**	**50,26**	**50,48**	**50,53**	**50,73**	**50,18**	**50,25**
maxARCH	91,12	82,42	79,46	79,09	78,44	77,80	77,13	76,70	76,40	76,39	76,43	76,62	76,87	76,99	77,49	77,90	78,38	78,82	79,31	80,12	80,66	81,69

Table A.1 (continued)

	Forecast Horizon in days																					
	5	10	15	16	18	20	22	24	26	28	30	32	34	35	40	45	50	60	70	80	90	100
MedHASE																						
minPEC	**19,52**	**12,27**	**10,81**	**10,01**	**9,10**	**8,42**	**8,45**	**8,14**	**7,95**	**7,56**	**7,77**	**7,54**	**7,44**	**7,43**	**7,80**	**7,38**	**8,08**	**8,41**	**8,22**	**9,39**	**8,33**	**8,74**
maxPEC	21,91	15,01	11,88	11,16	11,31	10,54	10,39	10,35	10,25	10,01	10,66	10,39	10,27	10,22	10,77	10,96	10,95	11,10	11,97	11,28	11,29	11,94
minARCH	**19,53**	**12,50**	**10,08**	**9,82**	**9,69**	**9,02**	**9,24**	**9,43**	**9,22**	**8,51**	**8,85**	**8,42**	**8,43**	**8,86**	**8,45**	**8,40**	**8,72**	**8,36**	**8,57**	**8,38**	**8,80**	**8,33**
maxARCH	28,21	20,02	17,90	17,26	15,91	16,05	15,27	15,08	15,30	14,95	14,90	15,31	15,77	15,53	15,46	15,09	15,92	16,42	19,19	22,97	27,93	36,67
MedHAAE																						
minPEC	**44,19**	**35,03**	**32,88**	**31,64**	**30,17**	**29,02**	**29,07**	**28,54**	**28,19**	**27,49**	**27,88**	**27,46**	**27,27**	**27,25**	**27,93**	**27,17**	**28,43**	**29,01**	**28,67**	**30,65**	**28,87**	**29,57**
maxPEC	46,80	38,75	34,47	33,41	33,63	32,47	32,23	32,17	32,02	31,63	32,65	32,24	32,05	31,98	32,82	33,10	33,10	33,31	34,60	33,58	33,61	34,55
minARCH	**44,20**	**35,35**	**31,75**	**31,33**	**31,12**	**30,04**	**30,39**	**30,70**	**30,37**	**29,17**	**29,74**	**29,02**	**29,03**	**29,77**	**29,07**	**28,97**	**29,52**	**28,91**	**29,28**	**28,95**	**29,67**	**28,86**
maxARCH	53,11	44,75	42,31	41,55	39,89	40,07	39,07	38,83	39,11	38,67	38,60	39,13	39,71	39,41	39,32	38,85	39,90	40,52	43,81	47,93	52,85	60,55
MeanLE																						
minPEC	**68,24**	**39,93**	**31,75**	**30,99**	**29,04**	**27,71**	**27,07**	**26,43**	**26,07**	**25,85**	**25,66**	**25,40**	**25,24**	**25,16**	**24,94**	**24,85**	**24,84**	**25,09**	**25,16**	**25,22**	**25,22**	**25,55**
maxPEC	77,35	46,44	37,00	35,93	33,76	32,67	32,14	31,50	31,09	30,71	30,34	30,02	29,86	29,72	29,16	28,93	28,89	28,89	28,37	28,03	27,70	27,74
minARCH	**63,02**	**40,18**	**33,82**	**33,10**	**31,36**	**29,90**	**29,03**	**28,31**	**27,70**	**27,19**	**26,84**	**26,42**	**26,21**	**26,06**	**25,40**	**24,77**	**24,41**	**23,99**	**23,40**	**23,01**	**22,44**	**22,27**
maxARCH	79,54	52,04	45,60	44,76	43,33	42,18	41,65	41,27	40,98	40,91	40,83	40,71	40,81	40,81	40,78	40,94	41,10	42,78	44,33	45,35	45,72	46,73
MedLE																						
minPEC	**22,80**	**14,22**	**10,94**	**10,65**	**10,04**	**8,65**	**9,27**	**8,49**	**8,02**	**8,19**	**7,59**	**7,72**	**8,17**	**8,33**	**8,36**	**8,20**	**9,10**	**9,06**	**9,62**	**9,73**	**9,52**	**9,14**
maxPEC	26,63	17,41	13,00	12,72	12,03	11,30	11,22	11,42	11,29	10,87	10,79	10,49	10,38	10,65	10,77	10,81	11,51	12,07	12,94	12,35	12,36	13,03
minARCH	**21,60**	**13,53**	**11,27**	**11,10**	**10,41**	**9,46**	**9,86**	**10,03**	**9,81**	**9,32**	**9,14**	**8,65**	**8,92**	**9,11**	**9,07**	**8,68**	**9,19**	**8,61**	**9,34**	**9,46**	**9,22**	**8,83**
maxARCH	32,39	22,40	18,06	17,53	16,90	16,38	15,43	15,85	15,30	14,79	14,31	13,37	13,68	14,18	14,83	15,71	17,07	18,56	21,65	21,97	23,58	26,19

Evaluation Criteria for the Conditional Standard Deviation, k=1

	Forecast Horizon in days																					
	5	10	15	16	18	20	22	24	26	28	30	32	34	35	40	45	50	60	70	80	90	100
MeanSE																						
minPEC	**0,535**	**0,398**	**0,335**	**0,323**	**0,301**	**0,288**	**0,277**	**0,271**	**0,267**	**0,265**	**0,263**	**0,260**	**0,259**	**0,259**	**0,255**	**0,250**	**0,245**	**0,239**	**0,234**	**0,227**	**0,220**	**0,216**
maxPEC	0,617	0,459	0,383	0,371	0,349	0,338	0,332	0,327	0,322	0,318	0,314	0,310	0,309	0,308	0,300	0,294	0,288	0,279	0,266	0,254	0,243	0,237
minARCH	**0,507**	**0,396**	**0,343**	**0,333**	**0,316**	**0,301**	**0,292**	**0,283**	**0,277**	**0,271**	**0,268**	**0,265**	**0,262**	**0,261**	**0,252**	**0,242**	**0,235**	**0,224**	**0,214**	**0,206**	**0,196**	**0,191**
maxARCH	0,656	0,513	0,448	0,435	0,414	0,400	0,392	0,386	0,380	0,375	0,371	0,367	0,364	0,363	0,358	0,353	0,347	0,345	0,343	0,341	0,334	0,333
MeanAE																						
minPEC	**4,944**	**4,111**	**3,759**	**3,698**	**3,583**	**3,504**	**3,466**	**3,448**	**3,431**	**3,416**	**3,388**	**3,381**	**3,386**	**3,393**	**3,394**	**3,410**	**3,433**	**3,464**	**3,466**	**3,484**	**3,493**	**3,491**
maxPEC	5,396	4,519	4,142	4,074	3,913	3,822	3,798	3,792	3,783	3,757	3,728	3,713	3,725	3,725	3,687	3,696	3,721	3,733	3,734	3,737	3,706	3,692
minARCH	**4,697**	**4,001**	**3,766**	**3,718**	**3,632**	**3,576**	**3,543**	**3,535**	**3,515**	**3,487**	**3,465**	**3,432**	**3,430**	**3,423**	**3,401**	**3,360**	**3,356**	**3,338**	**3,315**	**3,327**	**3,308**	**3,284**
maxARCH	5,460	4,762	4,483	4,428	4,337	4,261	4,235	4,228	4,203	4,184	4,166	4,156	4,173	4,180	4,194	4,229	4,268	4,356	4,443	4,497	4,507	4,549
MedSE																						
minPEC	**0,1322**	**0,0823**	**0,0739**	**0,0699**	**0,0597**	**0,0551**	**0,0560**	**0,0549**	**0,0538**	**0,0521**	**0,0499**	**0,0480**	**0,0474**	**0,0478**	**0,0498**	**0,0470**	**0,0490**	**0,0515**	**0,0531**	**0,0538**	**0,0542**	**0,0536**
maxPEC	0,1768	0,1218	0,0840	0,0806	0,0742	0,0696	0,0677	0,0665	0,0646	0,0605	0,0602	0,0613	0,0596	0,0628	0,0617	0,0647	0,0700	0,0689	0,0745	0,0717	0,0654	0,0685
minARCH	**0,1211**	**0,0783**	**0,0653**	**0,0632**	**0,0624**	**0,0596**	**0,0573**	**0,0575**	**0,0565**	**0,0536**	**0,0548**	**0,0499**	**0,0507**	**0,0523**	**0,0500**	**0,0516**	**0,0518**	**0,0500**	**0,0506**	**0,0531**	**0,0516**	**0,0486**
maxARCH	0,1693	0,1278	0,1077	0,1046	0,0967	0,0920	0,0929	0,0827	0,0860	0,0785	0,0819	0,0805	0,0859	0,0896	0,0864	0,0936	0,1001	0,1131	0,1401	0,1410	0,1492	0,1542
MedAE																						
minPEC	**3,636**	**2,868**	**2,719**	**2,645**	**2,444**	**2,347**	**2,367**	**2,344**	**2,319**	**2,282**	**2,234**	**2,190**	**2,177**	**2,186**	**2,232**	**2,168**	**2,213**	**2,269**	**2,304**	**2,319**	**2,327**	**2,315**
maxPEC	4,205	3,491	2,898	2,838	2,723	2,638	2,601	2,578	2,543	2,459	2,453	2,476	2,442	2,506	2,484	2,544	2,646	2,625	2,729	2,678	2,557	2,617
minARCH	**3,479**	**2,798**	**2,555**	**2,514**	**2,499**	**2,441**	**2,394**	**2,398**	**2,377**	**2,316**	**2,340**	**2,234**	**2,252**	**2,286**	**2,236**	**2,272**	**2,275**	**2,235**	**2,250**	**2,305**	**2,271**	**2,204**
maxARCH	4,115	3,576	3,282	3,234	3,110	3,034	3,047	2,876	2,932	2,802	2,861	2,837	2,931	2,993	2,939	3,059	3,163	3,362	3,743	3,755	3,863	3,927
MeanHASE																						
minPEC	**18,50**	**14,04**	**11,92**	**11,59**	**10,81**	**10,32**	**9,91**	**9,68**	**9,57**	**9,47**	**9,41**	**9,37**	**9,31**	**9,30**	**9,18**	**9,07**	**8,99**	**9,19**	**9,23**	**9,16**	**9,08**	**9,22**
maxPEC	19,98	15,57	13,59	13,38	13,00	12,68	12,77	12,68	12,62	12,48	12,31	12,19	12,06	11,99	11,65	11,45	11,26	11,21	10,85	10,53	10,26	10,25
minARCH	**20,34**	**15,77**	**13,49**	**13,06**	**12,25**	**11,68**	**11,34**	**11,04**	**10,82**	**10,64**	**10,52**	**10,45**	**10,32**	**10,25**	**9,91**	**9,57**	**9,30**	**9,05**	**8,74**	**8,45**	**8,11**	**8,03**
maxARCH	28,78	23,16	21,45	21,21	20,76	20,28	19,97	19,68	19,47	19,28	19,13	19,01	18,90	18,85	18,63	18,44	18,30	18,11	18,01	17,93	17,83	17,81

Table A.1 (continued)

	Forecast Horizon in days																					
	5	10	15	16	18	20	22	24	26	28	30	32	34	35	40	45	50	60	70	80	90	100
MeanHAAE																						
minPEC	**29,45**	**24,54**	**22,61**	**22,24**	**21,57**	**21,05**	**20,78**	**20,68**	**20,64**	**20,60**	**20,47**	**20,50**	**20,57**	**20,63**	**20,75**	**20,97**	**21,25**	**21,82**	**22,17**	**22,54**	**22,79**	**23,00**
maxPEC	30,68	25,95	24,17	24,00	23,64	23,45	23,41	23,41	23,39	23,24	23,07	23,01	23,08	23,08	22,90	23,06	23,31	23,62	23,82	23,97	24,09	24,24
minARCH	**30,46**	**26,09**	**24,45**	**24,12**	**23,53**	**23,09**	**22,82**	**22,72**	**22,58**	**22,41**	**22,24**	**22,05**	**21,98**	**21,95**	**21,83**	**21,66**	**21,66**	**21,64**	**21,59**	**21,71**	**21,58**	**21,52**
maxARCH	36,16	31,96	30,45	30,28	30,01	29,78	29,49	29,33	29,23	29,26	29,31	29,42	29,57	29,63	29,93	30,17	30,44	30,66	31,45	32,19	32,62	33,28
MedHASE																						
minPEC	**5,560**	**3,343**	**2,845**	**2,602**	**2,444**	**2,179**	**2,252**	**2,146**	**1,972**	**2,001**	**2,003**	**1,887**	**1,991**	**2,012**	**2,147**	**1,944**	**2,176**	**2,243**	**2,259**	**2,348**	**2,260**	**2,271**
maxPEC	6,158	4,088	3,167	2,888	3,001	2,757	2,702	2,759	2,645	2,724	2,678	2,676	2,552	2,618	2,773	2,718	2,767	2,940	3,195	2,951	2,913	3,147
minARCH	**5,366**	**3,329**	**2,523**	**2,652**	**2,564**	**2,372**	**2,441**	**2,485**	**2,444**	**2,270**	**2,247**	**2,173**	**2,269**	**2,273**	**2,198**	**2,124**	**2,264**	**2,144**	**2,242**	**2,356**	**2,185**	**2,124**
maxARCH	7,855	5,423	4,480	4,480	4,143	4,020	3,904	3,863	3,960	3,813	3,723	3,698	3,733	3,664	3,711	3,899	3,961	4,259	5,022	5,826	6,754	7,586
MedHAAE																						
minPEC	**23,58**	**18,29**	**16,87**	**16,13**	**15,63**	**14,76**	**15,01**	**14,65**	**14,04**	**14,15**	**14,15**	**13,74**	**14,11**	**14,19**	**14,65**	**13,94**	**14,75**	**14,98**	**15,03**	**15,32**	**15,03**	**15,07**
maxPEC	24,82	20,22	17,80	16,99	17,32	16,60	16,44	16,61	16,26	16,51	16,36	16,36	15,98	16,18	16,65	16,49	16,63	17,15	17,88	17,18	17,07	17,74
minARCH	**23,17**	**18,25**	**15,89**	**16,29**	**16,01**	**15,40**	**15,62**	**15,76**	**15,63**	**15,07**	**14,99**	**14,74**	**15,06**	**15,08**	**14,83**	**14,57**	**15,05**	**14,64**	**14,97**	**15,35**	**14,78**	**14,57**
maxARCH	28,03	23,29	21,16	21,17	20,36	20,05	19,76	19,66	19,90	19,53	19,29	19,23	19,32	19,14	19,26	19,75	19,90	20,64	22,41	24,14	25,99	27,54
MeanLE																						
minPEC	**17,06**	**9,98**	**7,94**	**7,75**	**7,26**	**6,93**	**6,77**	**6,61**	**6,52**	**6,46**	**6,41**	**6,35**	**6,31**	**6,29**	**6,24**	**6,21**	**6,21**	**6,27**	**6,29**	**6,30**	**6,30**	**6,39**
maxPEC	19,34	11,61	9,25	8,98	8,44	8,17	8,03	7,88	7,77	7,68	7,59	7,51	7,47	7,43	7,29	7,23	7,22	7,22	7,09	7,01	6,92	6,94
minARCH	**15,75**	**10,04**	**8,46**	**8,27**	**7,84**	**7,47**	**7,26**	**7,08**	**6,92**	**6,80**	**6,71**	**6,61**	**6,55**	**6,52**	**6,35**	**6,19**	**6,10**	**6,00**	**5,85**	**5,75**	**5,61**	**5,57**
maxARCH	19,89	13,01	11,40	11,19	10,83	10,55	10,41	10,32	10,24	10,23	10,21	10,18	10,20	10,20	10,20	10,23	10,27	10,70	11,08	11,34	11,43	11,68
MedLE																						
minPEC	**5,701**	**3,555**	**2,734**	**2,661**	**2,510**	**2,162**	**2,319**	**2,123**	**2,006**	**2,049**	**1,896**	**1,930**	**2,043**	**2,083**	**2,089**	**2,050**	**2,276**	**2,264**	**2,404**	**2,432**	**2,380**	**2,285**
maxPEC	6,657	4,352	3,249	3,179	3,007	2,825	2,806	2,854	2,821	2,718	2,698	2,623	2,595	2,662	2,693	2,703	2,877	3,018	3,235	3,089	3,090	3,257
minARCH	**5,400**	**3,383**	**2,818**	**2,774**	**2,603**	**2,364**	**2,465**	**2,507**	**2,452**	**2,329**	**2,285**	**2,163**	**2,229**	**2,278**	**2,268**	**2,169**	**2,297**	**2,152**	**2,335**	**2,366**	**2,306**	**2,207**
maxARCH	8,099	5,599	4,514	4,382	4,224	4,096	3,857	3,963	3,825	3,698	3,577	3,344	3,420	3,546	3,708	3,928	4,267	4,639	5,412	5,491	5,894	6,548

Figure A.1a

The plots indicate whether the ARCH models selected by the PEC algorithm achieve the lowest value of the evaluation criterion, for a forecasting horizon ranging from one day to one hundred days ahead. The value 2 indicates that the ARCH model selected by the PEC algorithm achieves the lowest value for the corresponding criterion and the specific forecasting horizon. The value 1 indicates the opposite. The realized volatilty measure is expressed as in (4.1). The evaluation criteria are the mean and the median values of the functions defined by (4.3), (4.4), (4.5), (4.6) and (4.7).

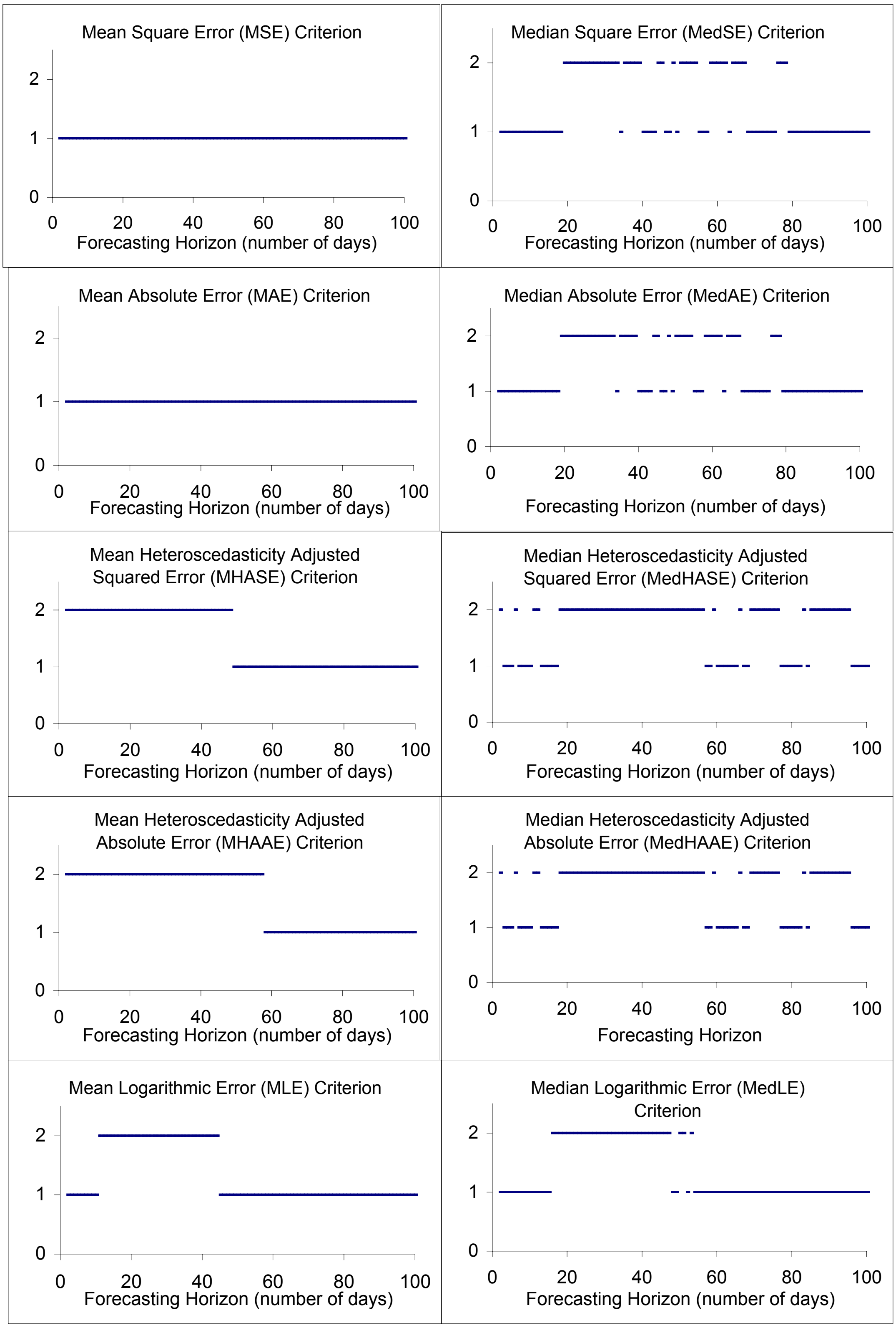

Figure A.1b

The plots indicate whether the ARCH models selected by the PEC algorithm achieve the lowest value of the evaluation criterion, for a forecasting horizon ranging from one day to one hundred days ahead. The value 2 indicates that the ARCH model selected by the PEC algorithm achieves the lowest value for the corresponding criterion and the specific forecasting horizon. The value 1 indicates the opposite. The realized volatilty measure is expressed as the square root of (4.1). The evaluation criteria are the mean and the median values of the functions defined by (4.3), (4.4), (4.5), (4.6) and (4.7).

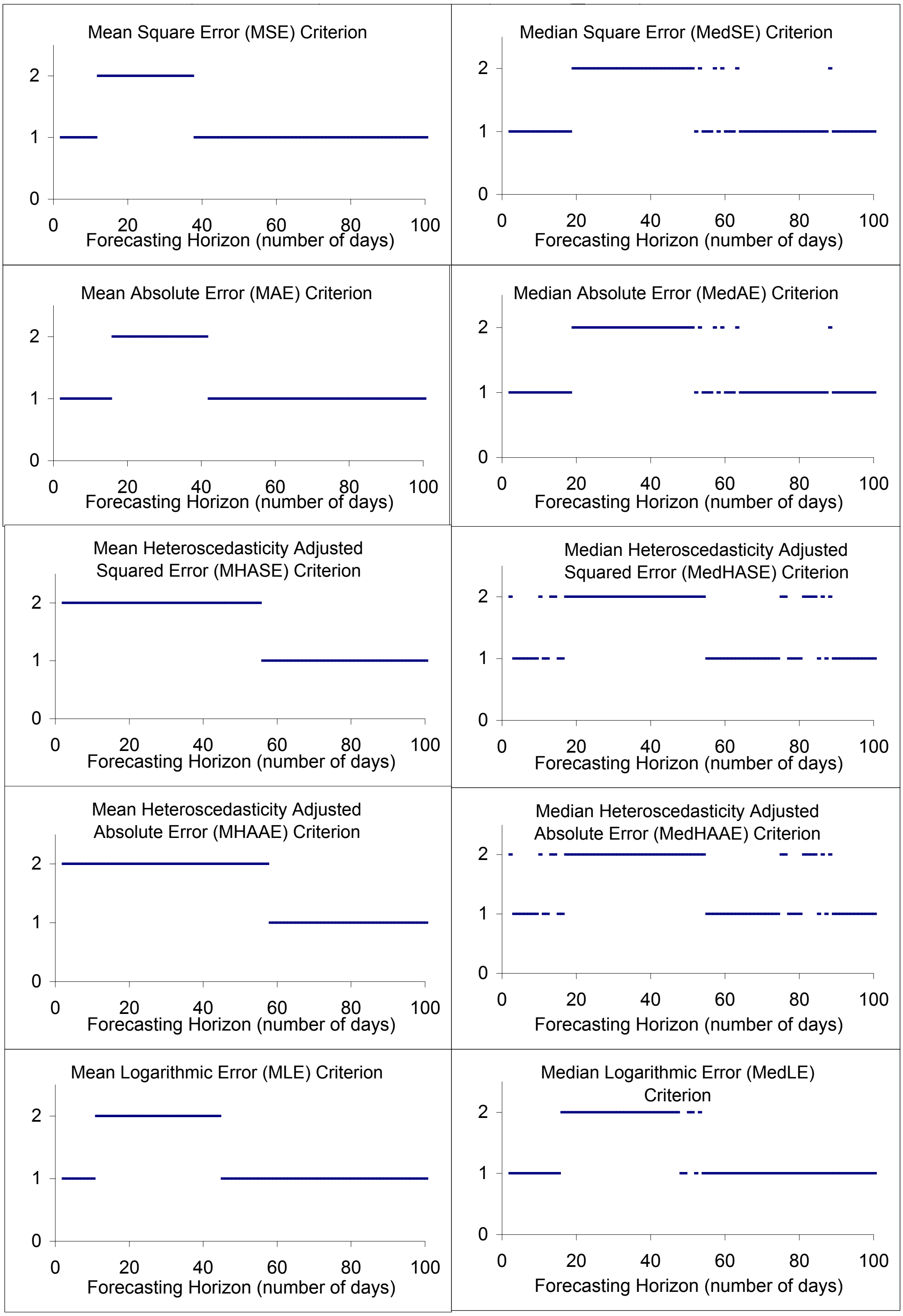

Table A.2

The percentage of times the ARCH models selected by the PEC algorithm perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	0%	0%	47%	56%	34%	26%	26%	54%	56%	34%
11-52	0%	0%	88%	100%	79%	62%	62%	100%	100%	79%
16-36	0%	0%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	40%	40%	65%	65%	35%	38%	38%	50%	50%	35%
11-52	64%	64%	88%	88%	83%	81%	81%	93%	93%	83%
16-36	86%	86%	95%	95%	100%	90%	90%	100%	100%	100%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.3

Average sample size for the PEC model selection algorithm suggested by the Evaluation Criteria for both the entire 16 to 36 day ahead forecasting horizon and for each day individually.

Forecasting Horizon (in number of days ahead)	Average sample size suggested by the Evaluation Criteria rating the performance of the PEC selection algorithm "best". Number of Criteria	Average sample size	Standard Deviation	Average sample size suggested by all the Evaluation Criteria considered. Number of Criteria	Average sample size	Standard Deviation
16-36	366	19,7	3,6	420	19,9	3,7
16	12	23,8	1,7	20	26,0	2,5
17	14	20,7	1,5	20	23,5	2,9
18	18	24,7	2,8	20	24,3	2,7
19	18	25,0	3,3	20	24,3	3,3
20	18	23,6	3,3	20	23,0	3,3
21	18	23,3	3,4	20	22,5	3,4
22	18	20,0	3,8	20	19,5	3,6
23	18	19,4	3,8	20	19,0	3,7
24	18	19,4	3,8	20	19,0	3,7
25	18	17,2	2,9	20	17,0	2,8
26	18	17,2	2,9	20	17,0	2,8
27	18	17,8	3,6	20	17,5	3,4
28	18	18,3	3,1	20	18,0	2,9
29	18	18,3	3,1	20	18,0	2,9
30	18	20,6	6,5	20	20,0	6,2
31	18	16,1	1,4	20	16,0	1,4
32	18	17,8	3,6	20	17,5	3,4
33	16	15,6	0,8	20	20,0	6,2
34	18	21,1	4,7	20	20,5	4,5
35	18	17,8	3,6	20	17,5	3,4
36	18	17,8	2,9	20	17,5	2,8

Table A.4.1

The percentage of times the ARCH models selected by the SEVar method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	2%	2%	1%	1%	3%	2%	2%	1%	1%	3%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	**Median**									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	4%	4%	2%	2%	3%	2%	2%	1%	1%	3%
11-52	2%	2%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	5%	5%	0%	0%	0%	0%	0%	0%	0%	0%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.4.2

The percentage of times the ARCH models selected by the AEVar method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	1%	2%	0%	0%	3%	2%	3%	0%	1%	3%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	**Median**									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	9%	9%	1%	1%	2%	6%	6%	1%	1%	2%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table A.4.3

The percentage of times the ARCH models selected by the SEDev method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	1%	3%	1%	0%	3%	2%	3%	1%	1%	3%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	**Median**									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	8%	8%	1%	1%	2%	5%	5%	1%	1%	2%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table A.4.4

The percentage of times the ARCH models selected by the AEDev method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	1%	3%	0%	0%	3%	2%	3%	0%	1%	3%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	9%	9%	1%	1%	2%	6%	6%	1%	1%	2%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.4.5

The percentage of times the ARCH models selected by the HASEVar method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	0%	0%	8%	16%	0%	0%	11%	12%	34%	0%
11-52	0%	0%	0%	14%	0%	0%	26%	5%	57%	0%
16-36	0%	0%	0%	5%	0%	0%	52%	0%	90%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	40%	40%	21%	21%	22%	26%	26%	20%	20%	22%
11-52	67%	67%	45%	45%	48%	57%	57%	45%	45%	48%
16-36	95%	95%	81%	81%	86%	86%	86%	81%	81%	86%

Table A.4.6

The percentage of times the ARCH models selected by the HAAEVar method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	2%	1%	4%	16%	0%	1%	12%	9%	34%	0%
11-52	0%	0%	0%	14%	0%	0%	29%	0%	57%	0%
16-36	0%	0%	0%	5%	0%	0%	57%	0%	90%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	36%	36%	26%	26%	24%	26%	26%	24%	24%	24%
11-52	64%	64%	52%	52%	52%	60%	60%	50%	50%	52%
16-36	90%	90%	100%	100%	100%	86%	86%	95%	95%	100%

Table A.4.7

The percentage of times the ARCH models selected by the HASEDev method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	1%	2%	3%	6%	0%	2%	2%	5%	8%	0%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	25%	25%	25%	25%	25%	21%	21%	27%	27%	25%
11-52	50%	50%	43%	43%	45%	48%	48%	48%	48%	45%
16-36	86%	86%	81%	81%	86%	76%	76%	90%	90%	86%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.4.8

The percentage of times the ARCH models selected by the HAAEDev method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	3%	2%	2%	2%	2%	2%	2%	2%	3%	2%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	3%	3%	8%	8%	6%	5%	5%	7%	7%	6%
11-52	5%	5%	5%	5%	5%	7%	7%	10%	10%	5%
16-36	10%	10%	10%	10%	10%	10%	10%	19%	19%	10%

Table A.4.9

The percentage of times the ARCH models selected by the LEVar method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	1%	2%	0%	0%	3%	2%	2%	0%	0%	3%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	8%	8%	1%	1%	2%	5%	5%	1%	1%	2%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table A.4.10

The percentage of times the ARCH models selected by the AIC method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	1%	1%	0%	0%	0%	0%	0%	0%	0%	0%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table A.4.11

The percentage of times the ARCH models selected by the SBC method perform "better" as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	0%	0%	0%	0%	0%	1%	1%	0%	0%	0%
11-52	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
16-36	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Table A.5.1

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the SEVar criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	96%	92%	100%	100%	95%	97%	94%	100%	99%	95%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	84%	84%	97%	97%	92%	85%	85%	95%	95%	92%
11-52	86%	86%	100%	100%	100%	88%	88%	100%	100%	100%
16-36	90%	90%	100%	100%	100%	95%	95%	100%	100%	100%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.5.2

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the AEVar criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	97%	94%	100%	100%	95%	97%	95%	100%	100%	95%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	88%	88%	99%	99%	96%	91%	91%	98%	98%	96%
11-52	95%	95%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	95%	95%	100%	100%	100%	100%	100%	100%	100%	100%

Table A.5.3

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the SEDev criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	96%	93%	100%	100%	95%	97%	95%	100%	100%	95%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	88%	88%	99%	99%	96%	91%	91%	98%	98%	96%
11-52	95%	95%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table A.5.4

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the AEDev criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	97%	93%	100%	100%	95%	97%	95%	100%	100%	95%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	87%	87%	98%	98%	96%	91%	91%	98%	98%	96%
11-52	93%	93%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	95%	95%	100%	100%	100%	100%	100%	100%	100%	100%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.5.5

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the HASEVar criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	93%	89%	100%	100%	94%	94%	91%	99%	98%	94%
11-52	100%	98%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	36%	36%	92%	92%	84%	48%	48%	90%	90%	84%
11-52	33%	33%	90%	90%	81%	55%	55%	90%	90%	81%
16-36	38%	38%	90%	90%	81%	52%	52%	90%	90%	81%

Table A.5.6

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the HAAEVar criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	0%	60%	99%	99%	93%	94%	89%	98%	96%	93%
11-52	0%	95%	100%	100%	100%	100%	98%	100%	100%	100%
16-36	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	36%	36%	92%	92%	83%	59%	57%	88%	88%	83%
11-52	26%	26%	93%	93%	79%	52%	52%	88%	88%	79%
16-36	19%	19%	90%	90%	76%	43%	43%	86%	86%	76%

Table A.5.7

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the HASEDev criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	82%	88%	100%	100%	93%	94%	91%	99%	98%	93%
11-52	100%	95%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	45%	45%	88%	88%	88%	60%	60%	87%	87%	88%
11-52	45%	45%	93%	93%	93%	60%	60%	93%	93%	93%
16-36	33%	33%	95%	95%	100%	43%	43%	100%	100%	100%

MSE: Mean Square Error
MAE: Mean Absolute Error
MHASE: Mean Heteroscedasticity Adjusted Squared Error
MHAAE: Mean Heteroscedasticity Adjusted Absolute Error
MLE: Mean Logarithmic Error
MedSE: Median Square Error
MedAE: Median Absolute Error
MedHASE: Median Heteroscedasticity Adjusted Squared Error
MedHAAE: Median Heteroscedasticity Adjusted Absolute Error
MedLE: Median Logarithmic Error

Table A.5.8

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the HAAEDev criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	30%	86%	100%	99%	95%	96%	92%	99%	98%	95%
11-52	71%	90%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	71%	71%	94%	94%	92%	80%	81%	93%	93%	92%
11-52	86%	86%	100%	100%	95%	86%	86%	100%	100%	95%
16-36	90%	90%	100%	100%	100%	95%	95%	100%	100%	100%

Table A.5.9

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the LEVar criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	97%	94%	100%	100%	96%	97%	95%	100%	100%	96%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	90%	90%	100%	100%	97%	92%	92%	99%	99%	97%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table A.5.10

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the AIC criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	99%	97%	100%	100%	96%	100%	96%	100%	100%	96%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	89%	89%	100%	100%	100%	93%	93%	99%	99%	100%
11-52	95%	95%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Table A.5.11

The percentage of times the ARCH models selected by the PEC method perform "better" than the ARCH models selected by the SBC criterion as judged by the evaluation criteria. The first and the second panel correspond to the mean and the median of the evaluation criteria, respectively. The left and the right part of the panels correspond to the volatility expressed as the variance and the standard deviation of the returns, respectively.

Days ahead forecasting horizon	Mean									
	Variance					Standard Deviation				
	MSE	MAE	MHASE	MHAAE	MLE	MSE	MAE	MHASE	MHAAE	MLE
1-100	98%	97%	100%	100%	96%	99%	96%	100%	100%	96%
11-52	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Days ahead forecasting horizon	Median									
	Variance					Standard Deviation				
	MedSE	MedAE	MedHASE	MedHAAE	MedLE	MedSE	MedAE	MedHASE	MedHAAE	MedLE
1-100	91%	91%	99%	99%	97%	92%	92%	99%	99%	97%
11-52	98%	98%	100%	100%	100%	100%	100%	100%	100%	100%
16-36	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%