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Research

A New Method for Constructing Confidence Intervals for the Index C_{pm}

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In the statistical literature on the study of the capability of processes through the use of indices, C_{pm} appears to have been one of the most widely used capability indices and its estimation has attracted much interest. In this article, a new method for constructing approximate confidence intervals or lower confidence limits for this index is suggested. The method is based on an approximation of the non-central chi-square distribution, which was proposed by Pearson. Its coverage appears to be more satisfactory compared with that achieved by any of the two most widely used methods that were proposed by Boyles, in situations where one is interested in assessing a lower confidence limit for C_{pm}. This is supported by the results of an extensive simulation study. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: process capability indices; non-central chi-square distribution; approximate confidence intervals; lower confidence limits; simulation study

1. INTRODUCTION

Process capability indices are used mainly in industry in order to measure the capability of a process to produce according to some specifications. A plethora of such indices has already been proposed in the last two decades and every year the appearance of new indices seems to be ceaseless (see, e.g., Gupta and Kotz[1], Yeh and Bhattacharya[2], Chen *et al.*[3], Borges and Ho[4], Perakis and Xekalaki[5] and Perakis and Xekalaki[6]). A review of these indices is provided in the textbooks by Kotz and Johnson[7] and Kotz and Lovelace[8] and the paper by Kotz and Johnson[9].

A large number of authors have dealt with the investigation of the properties and the estimation of such indices. In most of the cases it is assumed that the studied process is normally distributed and the observations produced through it are independent. Statistical inference on capability indices if any of these two basic assumptions is violated becomes a more difficult task and has attracted a small number of authors, such as Zhang[10] and Noorossana[11], who studied the case where the collected data are autocorrelated, and Clements[12], Pearn and Chen[13], Chen and Pearn[14], Tang and Than[15], Wu and Swain[16] and Chang *et al.*[17], who considered the case of non-normality.

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Among the suggested indices for normal processes, C_{pm} is, undoubtedly, one of the most widely used. It was initially introduced by Hsiang and Taguchi[18] and Chan *et al.*[19], and since then its properties and estimation techniques have also been investigated thoroughly by various other authors, such as Boyles[20], Pearn *et al.*[21], Subbaiah and Taam[22], Wright[23] and Zimmer *et al.*[24]. It is defined as

$$C_{\mathrm{pm}} = \frac{U-L}{6\sqrt{E(X-T)^2}} = \frac{U-L}{6\sqrt{\sigma^2+(\mu-T)^2}}$$

where L, U denote the lower and the upper specification limits, T corresponds to the target value and μ, σ refer to the mean and the standard deviation of the process, respectively.

Obviously, the assessment of the value of C_{pm} for a given process requires knowledge of both μ and σ. If these parameters are unknown, the value of the index has to be estimated. The two estimators of C_{pm} that appear most often in the literature are those proposed by Chan *et al.*[19] and Boyles[20], defined as

$$\tilde{C}_{\mathrm{pm}} = \frac{U-L}{6\sqrt{(n-1)^{-1}\sum_{i=1}^{n}(X_i-T)^2}} = \frac{U-L}{6\sqrt{S^2+n(n-1)^{-1}(\bar{X}-T)^2}} \tag{1}$$

and

$$\hat{C}_{\mathrm{pm}} = \frac{U-L}{6\sqrt{n^{-1}\sum_{i=1}^{n}(X_i-T)^2}} = \frac{U-L}{6\sqrt{(n-1)n^{-1}S^2+(\bar{X}-T)^2}} \tag{2}$$

respectively. In both (1) and (2), X_i, $i=1,\ldots,n$, denotes the elements of a random sample taken from the examined process, $\bar{X}$ is the sample mean and S^2 is the sample variance.

One may observe that estimators (1) and (2) differ in the type of the estimator used for the parameter $\sigma'^2=\sigma^2+(\mu-T)^2$. More specifically, in estimator (1) σ'^2 is estimated through

$$\tilde{\sigma}'^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i-T)^2 \tag{3}$$

while, in estimator (2), σ'^2 is estimated through

$$\hat{\sigma}'^2 = n^{-1}\sum_{i=1}^{n}(X_i-T)^2 \tag{4}$$

According to Boyles[20], the estimator given by (4) is an unbiased estimator of σ'^2 and its mean square error is smaller than that of the estimator given by (3). For this reason, Boyles[20] argues that estimator (2), which involves (4), is superior to (1). In contrast, as Kotz and Lovelace[8] point out, the bias and the mean square error of estimator (1) are smaller than those of (2). Subbaiah and Taam[22], based on simulation results, concluded that estimator (1) should be preferred for point estimation and estimator (2) is preferable when there is a need for assessing confidence intervals. It should be remarked that the statistical properties of the two estimators are quite similar, especially when the sample size is large, since $\hat{C}_{\mathrm{pm}} = \tilde{C}_{\mathrm{pm}}\sqrt{n/(n-1)}$.

Numerous techniques for obtaining approximate confidence intervals or merely lower confidence limits—since due to the fact that the very large values of a capability index are desirable, one usually is interested only in having a lower bound—have been proposed. Each of these techniques is based on a different approximation of the non-central chi-square distribution, which, as illustrated in the ensuing section, is involved in the distributions of (1) and (2). Among the suggested techniques, those that appear to perform better are the ones suggested by Boyles[20], as verified by Kushler and Hurley[25] and Subbaiah and Taam[22].

Boyles[20] considered two alternative approximations of the non-central chi-square distribution. The first is Patnaik's[26] approximation, which is an approximation of the form $c\chi_f^2$, where χ_f^2 denotes the chi-square distribution with f degrees of freedom and c, f are some constants. The use of this approximation results in the following $100(1-\alpha)\%$ approximate confidence interval and lower confidence limit for C_{pm}

$$\left(\hat{C}_{\mathrm{pm}}\sqrt{\chi^2_{\hat{f},\alpha/2}/\hat{f}},\ \hat{C}_{\mathrm{pm}}\sqrt{\chi^2_{\hat{f},1-\alpha/2}/\hat{f}}\right) \tag{5}$$

$$\hat{C}_{\mathrm{pm}}\sqrt{\chi^2_{\hat{f},\alpha}/\hat{f}} \tag{6}$$

where $\chi^2_{\hat{f},\alpha}$ denotes the $100\alpha\%$ percentile of the chi-square distribution with $\hat{f}$ degrees of freedom, $\hat{f} = n(1+\hat{\delta})^2/(1+2\hat{\delta})$, $\hat{\delta} = (\bar{X}-T)^2/\hat{\sigma}^2$ and $\hat{\sigma}^2 = S^2(n-1)/n$.

The second approximation considered by Boyles[20] involves the standard normal distribution, and he suggests it instead of (6) when $\hat{f}$ exceeds 100. According to this method a $100(1-\alpha)\%$ approximate confidence interval for C_{pm} is given by

$$(\hat{C}_{\mathrm{pm}}(1 - z_{1-\alpha/2}(2\hat{f})^{-1/2}),\ \hat{C}_{\mathrm{pm}}(1 + z_{1-\alpha/2}(2\hat{f})^{-1/2})) \tag{7}$$

while a lower confidence limit is given by

$$\hat{C}_{\mathrm{pm}}(1 - z_{1-\alpha}(2\hat{f})^{-1/2}) \tag{8}$$

In both of these relationships, z_α denotes the $100\alpha\%$ percentile of the standard normal distribution. Kushler and Hurley[25] suggest the use of lower confidence limit (6) and Subbaiah and Taam[22] found through simulation that confidence interval (7) is preferable to (5).

In this paper, an alternative approach for assessing approximate confidence limits for C_{pm} is proposed. The suggested technique, which is presented in Section 2, is based on an approximation of the non-central chi-square distribution initially suggested by Pearson[27]. As demonstrated in Section 3, where the performances of the new and the most commonly used of the existing techniques are compared via simulation, in most of the cases the suggested technique attains coverage quite close to the nominal and constitutes a rather useful tool, especially in situations where one wants to assess a lower confidence limit for C_{pm} and the sample size is not very large.

2. *THE SUGGESTED METHOD*

As already noted in the previous section, the distribution of $\hat{C}_{\mathrm{pm}}$ is related to the non-central chi-square distribution. Actually, as Boyles[20] points out, the distribution of $n\hat{\sigma}'^2/\sigma^2$ is the non-central chi-square with n degrees of freedom and non-centrality parameter $n\delta$, where $\delta = [(\mu - T)/\sigma]^2$. Therefore, it follows that

$$P\left(\chi'^2_{n,\alpha/2}(n\delta) < \frac{n\hat{\sigma}'^2}{\sigma^2} < \chi'^2_{n,1-\alpha/2}(n\delta)\right) = 1-\alpha \tag{9}$$

where $\chi'^2_{n,\alpha}(n\delta)$ denotes the $100\alpha\%$ percentile of the non-central chi-square distribution with n degrees of freedom and non-centrality parameter $n\delta$.

In the following, we propose a new method for constructing confidence limits for C_{pm}, which is based on an approximation of the non-central chi-square distribution suggested by Pearson[27]. In particular, Pearson[27] considered an improvement of Patnaik's[26] approximation by which the non-central chi-square distribution with ν degrees of freedom and non-centrality parameter λ is approximated by a distribution of the form $c\chi^2_f + b$, where χ^2_f denotes the chi-square distribution with f degrees of freedom and c, f, and b are some constants. The values of the constants c, f, and b are obtained by equating the first three moments of the non-central chi-square distribution and $c\chi^2_f + b$. Using the formulae for the rth crude moments of the chi-square distribution given by

$$\nu(\nu+2)\cdots[\nu+2(r-1)]$$

and the non-central chi-square distribution given by

$$2^r\Gamma\left(r+\frac{\nu}{2}\right)\sum_{j=0}^{r}\binom{r}{j}\frac{(\lambda/2)^j}{\Gamma(j+\nu/2)}$$

it can be found that the appropriate values of c, f and b are given by $(\nu+3\lambda)/(\nu+2\lambda)$, $(\nu+2\lambda)^3/(\nu+3\lambda)^2$ and $-\lambda^2/(\nu+3\lambda)$, respectively (see, e.g., Johnson and Pearson[28]). As Johnson *et al.*[29] point out, this approximation is better than that proposed by Patnaik[26], provided that the value at which one wants to assess the cumulative distribution function of the non-central chi-square distribution is large enough. Moreover, Johnson *et al.*[29] provide a table (Table 29.2 in their book), which compares the accuracy of the two approximations and reveals the superiority of that given by Pearson[27].

In the construction of confidence limits for C_{pm}, the non-central chi-square distribution that has to be approximated as n degrees of freedom and non-centrality parameter $n\delta$ and thus the values of c, f and b can be simplified to

$$c=\frac{1+3\delta}{1+2\delta} \tag{10}$$

$$f=\frac{n(1+2\delta)}{c^2} \tag{11}$$

and

$$b=-\frac{n\delta^2}{1+3\delta} \tag{12}$$

respectively. Hence, the left-hand side of (9) can be approximated by

$$P\left(c\chi^2_{f,\alpha/2}+b<\frac{n\hat{\sigma}'^2}{\sigma^2}<c\chi^2_{f,1-\alpha/2}+b\right)$$

where c, f and b are defined as in (10), (11) and (12), respectively. Taking into account the fact that $\sigma^2(1+\delta)=\sigma'^2$ and

$$\frac{\hat{\sigma}'^2}{\sigma'^2}=\frac{C^2_{\mathrm{pm}}}{\hat{C}^2_{\mathrm{pm}}}$$

one obtains, after some algebra, a $100(1-\alpha)\%$ approximate confidence interval for C_{pm} given by

$$\left(\hat{C}_{\mathrm{pm}}\sqrt{\frac{\hat{c}\chi_{\hat{f},\alpha/2}+\hat{b}}{n(1+\hat{\delta})}},\quad \hat{C}_{\mathrm{pm}}\sqrt{\frac{\hat{c}\chi^2_{\hat{f},1-\alpha/2}+\hat{b}}{n(1+\hat{\delta})}}\right) \tag{13}$$

where $\hat{c}$, $\hat{f}$ and $\hat{b}$ arise from (10), (11) and (12) substituting $\hat{\delta}$ for δ. Here, $\hat{\delta}$ can be either

$$\hat{\delta}_1=\left(\frac{\bar{X}-T}{\hat{\sigma}}\right)^2 \tag{14}$$

or

$$\hat{\delta}_2=\left(\frac{\bar{X}-T}{S}\right)^2 \tag{15}$$

Similarly, a $100(1-\alpha)\%$ approximate lower confidence limit for C_{pm} is given by

$$\hat{C}_{\mathrm{pm}}\sqrt{\frac{\hat{c}\chi^2_{\hat{f},\alpha}+\hat{b}}{n(1+\hat{\delta})}} \tag{16}$$

3. *A SIMULATION STUDY*

In order to compare the performance of the constructed confidence interval in (13) and the obtained lower confidence limit in (16) to those proposed by Boyles[20] (i.e. to confidence intervals (5) and (7) and to lower confidence limits (6) and (8)), a simulation study was conducted. In this study, random samples of sizes 20, 50 and 100 were generated from the normal distribution with a plethora of combinations of μ, σ and C_{pm}. The selected parameters have been chosen so as to investigate the performance of the methods for a wide range of index values and for on-target or off-target processes. The specification limits were assumed to be, without loss of generality, $L = 10$, $T = 15$ and $U = 20$. For each combination, 150 000 random samples were generated and, for each of these samples, the corresponding confidence intervals and lower confidence limits were assessed using all of the methods described above. The proportion of times that each of these limits contains the actual value of the index was recorded. Moreover, in all the cases the mean range of the obtained confidence intervals was assessed for each method.

The obtained results are summarized in Tables I–IV. More specifically, Tables I and II present the observed coverage (OC) and the mean range (MR) of the 90% and the 95% confidence intervals as well as the OC of the lower confidence limits, when the value of δ is estimated via (14). By contrast, Tables III and IV present the corresponding values when the value of δ is estimated through (15). Each row contains the values of μ and σ, the corresponding value of C_{pm}, the OC of the confidence intervals (13) (first entry), (5) (second entry) and (7) (third entry), the mean ranges of these confidence intervals and the observed coverage of the lower confidence limits (16), (6) and (8).

The basic conclusions that may be drawn from Tables I–IV are outlined in the sequel.

- The performance of the suggested lower confidence limit (16) appears to be better than those of (6) and (8), especially when the sample size is small. As the sample size increases the performances of (16) and (6) tend to become equivalent in most of the cases. For all the sample sizes considered, however, the superiority of (16) over (8) is apparent.
- The performance of confidence interval (7) appears to be better than that of confidence intervals (13) and (5), especially for small samples.
- The mean range of confidence interval (13) seems to be generally greater than that of (5), but smaller than that of (7).
- The choice of the estimator of δ does not appear to affect the coverage.

The first two conclusions can also be established from Table V, which summarizes the number of parameter combinations for which the new method performs better or worse than the two methods of Boyles[20]. The entries of Table V are of the form

$$f_{(i)} - f_{(j)}$$

and refer to the numbers $f_{(i)}$ and $f_{(j)}$ of times the confidence limits (i) and (j) achieve a coverage closer to the nominal. Since, sometimes the differences between the observed coverages are very small and the entries of Table V may become misleading, the same table also provides (in the brackets) the number of times where the observed coverage of each method is more proximal to the nominal coverage by a proportion greater than 0.1%. So, for example, 24–0, means that if $n = 20$, δ is estimated via $\hat{\delta}_1$ and the confidence level is 0.9, interval (13) leads to a coverage closer to the nominal than that of (5) in 24 parameter combinations, while the coverage of (5) is not closer to the nominal in any combination (this can be verified from Table I). Moreover, in the same case at only one time does the proximity of the observed coverage of (13) to the nominal exceed the corresponding proximity of (5) by a proportion greater than 0.1%. It should be noted that the reason why the sum of the values of some entries is not equal to the total number of the examined parameter combinations is that sometimes two or more methods result in the same observed coverage (such cases are not taken into account in the entries of Table V).

In conclusion, it appears from the results of the simulation study that the use of confidence interval (7) would be advisable in situations where one needs to assess a confidence interval for the actual value of the index C_{pm}, while for the assessment of a lower confidence limit formula (16) would be a preferable choice, especially when the sample size is not very large.

Table I. Observed coverage of 90% confidence limits using $\hat{\delta}_1$

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15	0.65	2.564	0.8985	1.3739	0.8975	0.8999	0.8534	0.9013	0.8998	0.6002	0.9003
			0.8984	1.3736	0.8974	0.8999	0.8534	0.9013	0.8997	0.6002	0.9003
			0.8997	1.3828	0.8865	0.9008	0.8557	0.8945	0.9000	0.6009	0.8955
15	0.75	2.222	0.8999	1.1903	0.8991	0.9022	0.7398	0.9004	0.8995	0.5201	0.8994
			0.8997	1.1900	0.8989	0.9022	0.7398	0.9004	0.8995	0.5201	0.8994
			0.9012	1.1980	0.8882	0.9025	0.7417	0.8938	0.8997	0.5208	0.8946
15	0.85	1.961	0.8997	1.0501	0.8992	0.8994	0.6527	0.9002	0.9003	0.4588	0.9006
			0.8996	1.0498	0.8991	0.8994	0.6527	0.9002	0.9003	0.4588	0.9006
			0.9010	1.0569	0.8885	0.8999	0.6544	0.8933	0.9009	0.4594	0.8960
15	1	1.667	0.8997	0.8925	0.8986	0.8990	0.5548	0.8999	0.8997	0.3900	0.8996
			0.8996	0.8923	0.8984	0.8990	0.5548	0.8998	0.8997	0.3900	0.8996
			0.9014	0.8983	0.8877	0.8995	0.5562	0.8935	0.8999	0.3906	0.8948
15	1.5	1.111	0.8988	0.5950	0.8979	0.9005	0.3701	0.8986	0.8999	0.2601	0.8993
			0.8987	0.5948	0.8978	0.9005	0.3700	0.8986	0.8999	0.2601	0.8992
			0.9002	0.5988	0.8871	0.9010	0.3710	0.8920	0.9000	0.2604	0.8944
15	2	0.833	0.8983	0.4462	0.8987	0.9007	0.2774	0.9003	0.8986	0.1950	08986
			0.8982	0.4460	0.8986	0.9007	0.2774	0.9003	0.8986	0.1950	0.8986
			0.8999	0.4491	0.8881	0.9012	0.2781	0.8935	0.8988	0.1953	0.8942
15.25	0.65	2.393	0.8952	1.2592	0.8985	0.8973	0.7865	0.8998	0.8980	0.5541	0.8984
			0.8950	1.2577	0.8980	0.8972	0.7862	0.8996	0.8979	0.5540	0.8983
			0.8961	1.2659	0.8872	0.8979	0.7882	0.8930	0.8980	0.5547	0.8941
15.25	0.75	2.108	0.8952	1.1141	0.8982	0.8979	0.6956	0.8992	0.9000	0.4898	0.9006
			0.8949	1.1131	0.8978	0.8979	0.6954	0.8991	0.9000	0.4897	0.9005
			0.8961	1.1204	0.8870	0.8985	0.6972	0.8925	0.9003	0.4903	0.8956
15.25	0.85	1.881	0.8973	0.9976	0.8978	0.8968	0.6223	0.8992	0.8981	0.4380	0.9001
			0.8972	0.9968	0.8975	0.8968	0.6221	0.8991	0.8981	0.4380	0.9001
			0.8986	1.0034	0.8869	0.8975	0.6237	0.8922	0.8983	0.4385	0.8957
15.25	1	1.617	0.8947	0.8592	0.8992	0.8992	0.5358	0.8994	0.8987	0.3772	0.8988
			0.8945	0.8586	0.8989	0.8992	0.5357	0.8994	0.8987	0.3772	0.8988
			0.8966	0.8643	0.8877	0.8999	0.5371	0.8927	0.8992	0.3777	0.8941
15.25	1.5	1.096	0.8992	0.5850	0.8989	0.9019	0.3644	0.9001	0.8999	0.2563	0.8983
			0.8990	0.5847	0.8987	0.9018	0.3644	0.9001	0.8999	0.2563	0.8983
			0.9006	0.5886	0.8879	0.9025	0.3653	0.8932	0.9001	0.2566	0.8938
15.25	2	0.827	0.8957	0.4419	0.8985	0.9011	0.2751	0.9001	0.9005	0.1935	0.8995
			0.8956	0.4417	0.8984	0.9011	0.2750	0.9001	0.9005	0.1935	0.8995
			0.8972	0.4447	0.8878	0.9017	0.2758	0.8935	0.9007	0.1937	0.8947

Table I. (Continued)

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15.5	0.65	2.032	0.8890	0.9926	0.8996	0.8967	0.6231	0.9017	0.8963	0.4398	0.9000
			0.8884	0.9895	0.8982	0.8966	0.6223	0.9011	0.8963	0.4395	0.8996
			0.8893	0.9951	0.8877	0.8974	0.6237	0.8950	0.8964	0.4400	0.8952
15.5	0.75	1.849	0.8903	0.9266	0.8973	0.8957	0.5817	0.9001	0.8976	0.4102	0.8994
			0.8896	0.9242	0.8961	0.8956	0.5811	0.8996	0.8976	0.4100	0.8991
			0.8908	0.9296	0.8862	0.8963	0.5824	0.8933	0.8979	0.4105	0.8946
15.5	1	1.491	0.8922	0.7713	0.8986	0.8974	0.4831	0.9004	0.8983	0.3407	0.9007
			0.8918	0.7700	0.8978	0.8973	0.4828	0.9002	0.8984	0.3406	0.9006
			0.8934	0.7748	0.8870	0.8979	0.4840	0.8939	0.8986	0.3410	0.8959
15.5	1.5	1.054	0.8935	0.5572	0.8983	0.8974	0.3478	0.8994	0.8989	0.2450	0.8999
			0.8933	0.5567	0.8980	0.8975	0.3478	0.8993	0.8989	0.2449	0.8998
			0.8951	0.5603	0.8865	0.8980	0.3487	0.8929	0.8990	0.2452	0.8954
15.5	2	0.808	0.8980	0.4299	0.8986	0.9002	0.2682	0.8979	0.8988	0.1886	0.9008
			0.8979	0.4296	0.8984	0.9002	0.2682	0.8979	0.8989	0.1886	0.9008
			0.8993	0.4325	0.8873	0.9006	0.2689	0.8912	0.8995	0.1888	0.8961
16	0.15	1.648	0.8740	0.1727	0.8892	0.8909	0.1116	0.8974	0.8964	0.0795	0.8970
			0.8737	0.1727	0.8887	0.8906	0.1115	0.8974	0.8964	0.0794	0.8972
			0.8737	0.1727	0.8872	0.8907	0.1115	0.8970	0.8964	0.0794	0.8977
16	0.25	1.617	0.8772	0.2757	0.8945	0.8909	0.1776	0.8990	0.8957	0.1264	0.9002
			0.8768	0.2753	0.8935	0.8906	0.1775	0.8985	0.8956	0.1263	0.9001
			0.8768	0.2755	0.8903	0.8907	0.1775	0.8968	0.8956	0.1264	0.8994
16	0.5	1.491	0.8817	0.4597	0.8983	0.8926	0.2928	0.9028	0.8951	0.2075	0.9026
			0.8805	0.4580	0.8962	0.8921	0.2924	0.9018	0.8950	0.2074	0.9020
			0.8808	0.4591	0.8897	0.8923	0.2927	0.8980	0.8952	0.2075	0.8995
16	1	1.179	0.8885	0.5352	0.8999	0.8955	0.3366	0.9033	0.8978	0.2377	0.9025
			0.8875	0.5330	0.8980	0.8953	0.3361	0.9023	0.8977	0.2375	0.9020
			0.8885	0.5357	0.8886	0.8956	0.3367	0.8964	0.8980	0.2377	0.8984
16	1.5	0.925	0.8915	0.4635	0.8990	0.8964	0.2907	0.9016	0.8992	0.2052	0.8995
			0.8909	0.4623	0.8979	0.8962	0.2904	0.9012	0.8993	0.2051	0.8992
			0.8921	0.4651	0.8875	0.8966	0.2911	0.8949	0.8993	0.2053	0.8950
16	2	0.745	0.8930	0.3857	0.8987	0.8959	0.2418	0.8983	0.8991	0.1704	0.9011
			0.8928	0.3851	0.8980	0.8958	0.2417	0.8980	0.8991	0.1703	0.9010
			0.8939	0.3875	0.8874	0.8965	0.2423	0.8914	0.8995	0.1705	0.8965
17	0.25	0.827	0.8758	0.0725	0.8903	0.8900	0.0469	0.8952	0.8942	0.0334	0.8964
			0.8757	0.0725	0.8901	0.8899	0.0469	0.8953	0.8942	0.0334	0.8968
			0.8757	0.0725	0.8890	0.8898	0.0469	0.8954	0.8941	0.0334	0.8979

Table II. Observed coverage of 95% confidence limits using $\hat{\delta}_1$

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15	0.65	2.564	0.9481	1.6342	0.9482	0.9502	1.0167	0.9501	0.9508	0.7149	0.9504
			0.9480	1.6337	0.9481	0.9502	1.0166	0.9501	0.9508	0.7149	0.9504
			0.9501	1.6474	0.9459	0.9510	1.0200	0.9484	0.9512	0.7160	0.9492
15	0.75	2.222	0.9492	1.4150	0.9498	0.9493	0.8813	0.9493	0.9502	0.6197	0.9504
			0.9492	1.4146	0.9496	0.9493	0.8813	0.9492	0.9502	0.6197	0.9504
			0.9511	1.4264	0.9472	0.9501	0.8841	0.9478	0.9506	0.6207	0.9492
15	0.85	1.961	0.9492	1.2490	0.9498	0.9490	0.7772	0.9501	0.9505	0.5466	0.9503
			0.9491	1.2486	0.9496	0.9489	0.7772	0.9501	0.9505	0.5466	0.9503
			0.9512	1.2590	0.9469	0.9497	0.7797	0.9482	0.9509	0.5474	0.9492
15	1	1.667	0.9492	1.0610	0.9493	0.9499	0.6610	0.9499	0.9505	0.4646	0.9505
			0.9491	1.0607	0.9492	0.9499	0.6610	0.9499	0.9505	0.4646	0.9505
			0.9512	1.0695	0.9467	0.9507	0.6632	0.9480	0.9510	0.4653	0.9494
15	1.5	1.111	0.9495	0.7078	0.9503	0.9502	0.4404	0.9505	0.9498	0.3097	0.9499
			0.9494	0.7076	0.9502	0.9502	0.4404	0.9505	0.9498	0.3097	0.9499
			0.9512	0.7135	0.9475	0.9509	0.4418	0.9485	0.9502	0.3102	0.9487
15	2	0.833	0.9492	0.5311	0.9493	0.9496	0.3305	0.9491	0.9503	0.2323	0.9503
			0.9492	0.5309	0.9492	0.9496	0.3305	0.9491	0.9503	0.2323	0.9503
			0.9511	0.5353	0.9466	0.9503	0.3316	0.9473	0.9506	0.2326	0.9492
15.25	0.65	2.393	0.9448	1.4972	0.9490	0.9480	0.9369	0.9489	0.9492	0.6601	0.9500
			0.9447	1.4951	0.9485	0.9481	0.9364	0.9488	0.9492	0.6600	0.9499
			0.9466	1.5071	0.9462	0.9488	0.9394	0.9472	0.9496	0.6611	0.9486
15.25	0.75	2.108	0.9459	1.325	0.9493	0.9490	0.8287	0.9495	0.9498	0.5836	0.9504
			0.9458	1.323	0.9488	0.9490	0.8285	0.9493	0.9498	0.5835	0.9503
			0.9478	1.334	0.9462	0.9497	0.8311	0.9476	0.9502	0.5845	0.9491
15.25	0.85	1.881	0.9474	1.1865	0.9495	0.9488	0.7408	0.9497	0.9495	0.5218	0.9500
			0.9473	1.1853	0.9492	0.9488	0.7407	0.9496	0.9495	0.5218	0.9499
			0.9493	1.1950	0.9466	0.9496	0.7430	0.9479	0.9498	0.5226	0.9486
15.25	1	1.617	0.9483	1.0227	0.9499	0.9488	0.6386	0.9491	0.9490	0.4494	0.9498
			0.9482	1.0219	0.9495	0.9489	0.6385	0.9490	0.9490	0.4493	0.9497
			0.9501	1.0303	0.9471	0.9495	0.6405	0.9473	0.9494	0.4500	0.9486
15.25	1.5	1.096	0.9479	0.6961	0.9483	0.9505	0.4336	0.9506	0.9497	0.3052	0.9498
			0.9478	0.6957	0.9481	0.9505	0.4336	0.9506	0.9497	0.3052	0.9498
			0.9497	0.7015	0.9456	0.9512	0.4350	0.9489	0.9501	0.3057	0.9485
15.25	2	0.827	0.9476	0.5257	0.9490	0.9501	0.3276	0.9506	0.9502	0.2304	0.9500
			0.9475	0.5255	0.9488	0.9501	0.3275	0.9506	0.9501	0.2304	0.9500
			0.9495	0.5299	0.9463	0.9509	0.3286	0.9489	0.9505	0.2308	0.9487

Table II. (Continued)

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15.5	0.65	2.032	0.9413	1.1812	0.9508	0.9462	0.7423	0.9522	0.9487	0.5240	0.9520
			0.9410	1.1764	0.9491	0.9461	0.7412	0.9513	0.9487	0.5236	0.9514
			0.9426	1.1847	0.9469	0.9468	0.7432	0.9497	0.9490	0.5243	0.9502
15.5	0.75	1.849	0.9416	1.1039	0.9489	0.9469	0.6928	0.9507	0.9491	0.4886	0.9508
			0.9414	1.1002	0.9475	0.9468	0.6919	0.9500	0.9492	0.4883	0.9503
			0.9432	1.1084	0.9449	0.9476	0.6939	0.9485	0.9495	0.4890	0.9490
15.5	1	1.491	0.9439	0.9189	0.9485	0.9479	0.5760	0.9487	0.9488	0.4057	0.9500
			0.9439	0.9169	0.9478	0.9480	0.5755	0.9483	0.9488	0.4056	0.9498
			0.9457	0.9241	0.9451	0.9487	0.5773	0.9468	0.9490	0.4062	0.9487
15.5	1.5	1.054	0.9457	0.6629	0.9485	0.9482	0.4142	0.9492	0.9490	0.2918	0.9499
			0.9457	0.6621	0.9481	0.9482	0.4141	0.9490	0.9490	0.2918	0.9498
			0.9477	0.6675	0.9455	0.9488	0.4154	0.9473	0.9494	0.2922	0.9485
15.5	2	0.808	0.9476	0.5114	0.9491	0.9489	0.3193	0.9490	0.9495	0.2247	0.9491
			0.9476	0.5110	0.9487	0.9489	0.3192	0.9489	0.9495	0.2247	0.9491
			0.9494	0.5152	0.9462	0.9496	0.3202	0.9473	0.9499	0.2251	0.9480
16	0.15	1.648	0.9287	0.2057	0.9411	0.9408	0.1329	0.9463	0.9467	0.0947	0.9480
			0.9286	0.2056	0.9403	0.9407	0.1328	0.9458	0.9466	0.0947	0.9479
			0.9285	0.2057	0.9401	0.9407	0.1328	0.9460	0.9466	0.0947	0.9485
16	0.25	1.617	0.9312	0.3288	0.9456	0.9423	0.2117	0.9496	0.9471	0.1506	0.9507
			0.9308	0.3283	0.9439	0.9423	0.2115	0.9487	0.9470	0.1506	0.9502
			0.9310	0.3286	0.9431	0.9423	0.2116	0.9483	0.9470	0.1506	0.9503
16	0.5	1.491	0.9360	0.5478	0.9513	0.9436	0.3492	0.9519	0.9468	0.2474	0.9518
			0.9349	0.5454	0.9482	0.9432	0.3486	0.9503	0.9466	0.2472	0.9508
			0.9355	0.5470	0.9467	0.9435	0.3490	0.9494	0.9467	0.2473	0.9503
16	1	1.179	0.9404	0.6375	0.9512	0.9461	0.4012	0.9525	0.9488	0.2833	0.9522
			0.9395	0.6343	0.9487	0.9460	0.4004	0.9510	0.9486	0.2830	0.9513
			0.9410	0.6382	0.9464	0.9465	0.4014	0.9494	0.9489	0.2834	0.9503
16	1.5	0.925	0.9426	0.5521	0.9492	0.9470	0.3464	0.9508	0.9486	0.2443	0.9512
			0.9425	0.5503	0.9477	0.9469	0.3460	0.9500	0.9485	0.2441	0.9508
			0.9441	0.5544	0.9453	0.9477	0.3470	0.9485	0.9488	0.2445	0.9496
16	2	0.745	0.9450	0.4592	0.9496	0.9476	0.2879	0.9503	0.9490	0.2030	0.9508
			0.9450	0.4582	0.9487	0.9477	0.2877	0.9498	0.9491	0.2029	0.9506
			0.9469	0.4618	0.9463	0.9484	0.2886	0.9482	0.9493	0.2032	0.9494
17	0.25	0.827	0.9292	0.0864	0.9399	0.9425	0.0558	0.9460	0.9467	0.0398	0.9471
			0.9290	0.0864	0.9393	0.9425	0.0558	0.9458	0.9467	0.0398	0.9472
			0.9291	0.0864	0.9393	0.9425	0.0558	0.9462	0.9468	0.0398	0.9480

Table III. Observed coverage of 90% confidence limits using $\hat{\delta}_2$

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15	0.65	2.564	0.9001	1.3737	0.8987	0.8990	0.8540	0.8989	0.8998	0.6001	0.8994
			0.8999	1.3734	0.8986	0.8991	0.8540	0.8989	0.8998	0.6001	0.8994
			0.9011	1.3826	0.8878	0.8996	0.8563	0.8920	0.9001	0.6009	0.8947
15	0.75	2.222	0.8990	1.1896	0.9000	0.8998	0.7399	0.8999	0.8998	0.5202	0.8994
			0.8990	1.1893	0.8999	0.8998	0.7399	0.8999	0.8998	0.5202	0.8994
			0.9002	1.1974	0.8891	0.9005	0.7418	0.8932	0.9001	0.5208	0.8949
15	0.85	1.961	0.8994	1.0503	0.8991	0.8999	0.6526	0.9012	0.9005	0.4590	0.9003
			0.8993	1.0501	0.8990	0.8999	0.6526	0.9012	0.9005	0.4590	0.9003
			0.9010	1.0572	0.8884	0.9006	0.6543	0.8945	0.9008	0.4596	0.8958
15	1	1.667	0.8992	0.8929	0.8988	0.8983	0.5549	0.8989	0.8982	0.3901	0.8988
			0.8991	0.8927	0.8988	0.8983	0.5549	0.8989	0.8982	0.3901	0.8988
			0.9005	0.8987	0.8885	0.8988	0.5563	0.8923	0.8985	0.3906	0.8941
15	1.5	1.111	0.8986	0.5944	0.9005	0.9012	0.3699	0.9000	0.9003	0.2600	0.8997
			0.8985	0.5943	0.9004	0.9011	0.3699	0.9000	0.9003	0.2600	0.8997
			0.9005	0.5983	0.8896	0.9012	0.3709	0.8930	0.9007	0.2604	0.8948
15	2	0.833	0.8988	0.4462	0.8988	0.8996	0.2776	0.8985	0.8999	0.1951	0.8992
			0.8988	0.4461	0.8988	0.8996	0.2776	0.8985	0.8999	0.1951	0.8992
			0.9008	0.4491	0.8882	0.9000	0.2783	0.8919	0.9003	0.1953	0.8949
15.25	0.65	2.393	0.8946	1.2598	0.8982	0.8993	0.7869	0.8996	0.9000	0.5543	0.8993
			0.8944	1.2584	0.8978	0.8993	0.7866	0.8994	0.8999	0.5542	0.8992
			0.8958	1.2666	0.8871	0.8997	0.7886	0.8929	0.9002	0.5549	0.8946
15.25	0.75	2.108	0.8962	1.1154	0.8989	0.8987	0.6954	0.9007	0.8970	0.4900	0.8983
			0.8961	1.1144	0.8986	0.8987	0.6953	0.9006	0.8971	0.4900	0.8982
			0.8977	1.1218	0.8881	0.8995	0.6971	0.8940	0.8975	0.4906	0.8939
15.25	0.85	1.881	0.8961	0.9970	0.9000	0.8979	0.6227	0.8987	0.8999	0.4382	0.8998
			0.8960	0.9963	0.8997	0.8979	0.6225	0.8986	0.8999	0.4381	0.8998
			0.8977	1.0029	0.8892	0.8984	0.6242	0.8916	0.9003	0.4387	0.8955
15.25	1	1.617	0.8958	0.8607	0.8985	0.8988	0.5361	0.8999	0.8998	0.3773	0.8988
			0.8956	0.8602	0.8984	0.8988	0.5360	0.8998	0.8998	0.3773	0.8987
			0.8971	0.8659	0.8880	0.8992	0.5374	0.8933	0.8999	0.3778	0.8942
15.25	1.5	1.096	0.8996	0.5855	0.8981	0.8980	0.3644	0.8978	0.8989	0.2563	0.8985
			0.8994	0.5853	0.8980	0.8980	0.3644	0.8978	0.8989	0.2563	0.8984
			0.9008	0.5892	0.8876	0.8986	0.3653	0.8909	0.8993	0.2566	0.8941
15.25	2	0.827	0.8990	0.4423	0.8996	0.8993	0.2750	0.9003	0.8999	0.1935	0.8989
			0.8989	0.4422	0.8995	0.8993	0.2750	0.9003	0.8999	0.1935	0.8989
			0.9006	0.4451	0.8889	0.9003	0.2757	0.8939	0.9001	0.1937	0.8943

Table III. (Continued)

			$n = 20$			$n = 50$			$n = 100$		
			OC	MR	OC	OC	MR	OC	OC	MR	OC
			(13)	(13)	(16)	(13)	(13)	(16)	(13)	(13)	(16)
			(5)	(5)	(6)	(5)	(5)	(6)	(5)	(5)	(6)
μ	σ	C_{pm}	(7)	(7)	(8)	(7)	(7)	(8)	(7)	(7)	(8)
15.5	0.65	2.032	0.8898	0.9977	0.8999	0.8970	0.6245	0.9029	0.8985	0.4403	0.9015
			0.8891	0.9946	0.8987	0.8967	0.6237	0.9022	0.8982	0.4400	0.9011
			0.8904	1.0003	0.8889	0.8973	0.6251	0.8959	0.8984	0.4405	0.8970
15.5	0.75	1.849	0.8929	0.9307	0.8996	0.8975	0.5822	0.9023	0.8983	0.4104	0.9010
			0.8924	0.9283	0.8985	0.8973	0.5816	0.9017	0.8983	0.4102	0.9007
			0.8931	0.9339	0.8879	0.8977	0.5830	0.8951	0.8985	0.4107	0.8964
15.5	1	1.491	0.8941	0.7730	0.8998	0.8977	0.4838	0.8993	0.8989	0.3409	0.9005
			0.8939	0.7717	0.8991	0.8976	0.4835	0.8990	0.8989	0.3408	0.9003
			0.8952	0.7766	0.8887	0.8980	0.4847	0.8923	0.8993	0.3412	0.8958
15.5	1.5	1.054	0.8950	0.5578	0.8979	0.8985	0.3479	0.8996	0.8987	0.2449	0.8987
			0.8950	0.5573	0.8976	0.8985	0.3478	0.8994	0.8988	0.2449	0.8986
			0.8966	0.5610	0.8873	0.8993	0.3487	0.8928	0.8989	0.2452	0.8942
15.5	2	0.808	0.8969	0.4300	0.8989	0.8988	0.2680	0.8995	0.8993	0.1887	0.8992
			0.8968	0.4298	0.8987	0.8988	0.2680	0.8994	0.8993	0.1887	0.8992
			0.8983	0.4326	0.8880	0.8992	0.2687	0.8932	0.8995	0.1889	0.8944
16	0.15	1.648	0.8848	0.1770	0.8963	0.8939	0.1127	0.8987	0.8979	0.0799	0.8992
			0.8844	0.1769	0.8959	0.8938	0.1127	0.8987	0.8978	0.0798	0.8993
			0.8843	0.1770	0.8944	0.8938	0.1127	0.8984	0.8978	0.0799	0.8998
16	0.25	1.617	0.8853	0.2822	0.8982	0.8944	0.1793	0.9018	0.8974	0.1270	0.9004
			0.8847	0.2818	0.8971	0.8943	0.1792	0.9014	0.8973	0.1270	0.9002
			0.8849	0.2821	0.8938	0.8944	0.1793	0.8997	0.8973	0.1270	0.8996
16	0.5	1.491	0.8896	0.4678	0.9035	0.8964	0.2950	0.9049	0.8979	0.2083	0.9045
			0.8881	0.4662	0.9013	0.8959	0.2946	0.9038	0.8976	0.2081	0.9039
			0.8885	0.4673	0.8950	0.8959	0.2949	0.8998	0.8977	0.2082	0.9015
16	1	1.179	0.8915	0.5391	0.9030	0.8962	0.3379	0.9030	0.8982	0.2382	0.9017
			0.8904	0.5369	0.9012	0.8958	0.3373	0.9019	0.8979	0.2380	0.9011
			0.8914	0.5396	0.8923	0.8961	0.3380	0.8958	0.8982	0.2383	0.8972
16	1.5	0.925	0.8920	0.4652	0.9001	0.8961	0.2913	0.8998	0.8970	0.2053	0.8989
			0.8917	0.4640	0.8990	0.8961	0.2910	0.8991	0.8970	0.2052	0.8985
			0.8930	0.4668	0.8888	0.8967	0.2917	0.8927	0.8971	0.2054	0.8941
16	2	0.745	0.8939	0.3868	0.8990	0.8980	0.2419	0.8997	0.8982	0.1704	0.9003
			0.8935	0.3862	0.8984	0.8981	0.2418	0.8995	0.8982	0.1704	0.9001
			0.8952	0.3886	0.8876	0.8986	0.2424	0.8929	0.8987	0.1706	0.8955
17	0.25	0.827	0.8854	0.0743	0.8963	0.8936	0.0473	0.8967	0.8965	0.0335	0.8989
			0.8853	0.0743	0.8961	0.8934	0.0473	0.8968	0.8965	0.0335	0.8993
			0.8853	0.0743	0.8951	0.8936	0.0473	0.8970	0.8964	0.0335	0.9004

Table IV. Observed coverage of 95% confidence limits using $\hat{\delta}_2$

			$n = 20$			$n = 50$			$n = 100$		
μ	σ	C_{pm}	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
15	0.65	2.564	0.9506	1.6337	0.9499	0.9490	1.0170	0.9497	0.9488	0.7148	0.9494
			0.9505	1.6332	0.9498	0.9490	1.0170	0.9497	0.9488	0.7148	0.9494
			0.9522	1.6469	0.9474	0.9497	1.0203	0.9479	0.9492	0.7160	0.9481
15	0.75	2.222	0.9491	1.4155	0.9496	0.9493	0.8814	0.9502	0.9492	0.6197	0.9485
			0.9491	1.4151	0.9495	0.9493	0.8814	0.9502	0.9492	0.6197	0.9485
			0.9511	1.4269	0.9474	0.9500	0.8843	0.9484	0.9495	0.6207	0.9473
15	0.85	1.961	0.9486	1.2494	0.9490	0.9488	0.7773	0.9491	0.9504	0.5467	0.9503
			0.9485	1.2491	0.9489	0.9488	0.7773	0.9491	0.9504	0.5467	0.9503
			0.9503	1.2595	0.9464	0.9495	0.7799	0.9473	0.9507	0.5475	0.9489
15	1	1.667	0.9489	1.0616	0.9494	0.9501	0.6605	0.9505	0.9504	0.4645	0.9501
			0.9488	1.0613	0.9493	0.9501	0.6605	0.9505	0.9504	0.4645	0.9501
			0.9506	1.0702	0.9470	0.9507	0.6626	0.9488	0.9508	0.4653	0.9488
15	1.5	1.111	0.9496	0.7078	0.9496	0.9502	0.4406	0.9499	0.9499	0.3098	0.9492
			0.9496	0.7076	0.9494	0.9502	0.4406	0.9499	0.9499	0.3098	0.9492
			0.9514	0.7135	0.9468	0.9509	0.4420	0.9480	0.9502	0.3103	0.9480
15	2	0.833	0.9484	0.5308	0.9495	0.9514	0.3305	0.9500	0.9495	0.2323	0.9500
			0.9483	0.5306	0.9493	0.9514	0.3304	0.9500	0.9495	0.2323	0.9500
			0.9502	0.5351	0.9469	0.9521	0.3315	0.9483	0.9499	0.2327	0.9489
15.25	0.65	2.393	0.9459	1.5007	0.9488	0.9474	0.9375	0.9493	0.9495	0.6599	0.9504
			0.9459	1.4986	0.9483	0.9476	0.9371	0.9491	0.9495	0.6598	0.9502
			0.9476	1.5107	0.9456	0.9483	0.9401	0.9471	0.9499	0.6608	0.9488
15.25	0.75	2.108	0.9461	1.3252	0.9492	0.9492	0.8290	0.9497	0.9493	0.5837	0.9489
			0.9461	1.3237	0.9487	0.9493	0.8287	0.9495	0.9494	0.5836	0.9488
			0.9481	1.3345	0.9463	0.9500	0.8314	0.9480	0.9497	0.5845	0.9477
15.25	0.85	1.881	0.9474	1.1877	0.9488	0.9484	0.7412	0.9499	0.9487	0.5219	0.9492
			0.9473	1.1866	0.9485	0.9484	0.7410	0.9498	0.9487	0.5219	0.9491
			0.9492	1.1964	0.9462	0.9491	0.7434	0.9480	0.9491	0.5227	0.9479
15.25	1	1.617	0.9475	1.0240	0.9489	0.9493	0.6385	0.9498	0.9490	0.4494	0.9486
			0.9475	1.0232	0.9486	0.9493	0.6384	0.9497	0.9490	0.4494	0.9486
			0.9494	1.0317	0.9461	0.9501	0.6405	0.9480	0.9493	0.4501	0.9472
15.25	1.5	1.096	0.9483	0.6969	0.9484	0.9496	0.4339	0.9501	0.9491	0.3052	0.9492
			0.9482	0.6966	0.9482	0.9496	0.4338	0.9501	0.9490	0.3052	0.9492
			0.9503	0.7024	0.9456	0.9505	0.4352	0.9483	0.9494	0.3057	0.9482
15.25	2	0.827	0.9489	0.5262	0.9499	0.9493	0.3277	0.9490	0.9500	0.2304	0.9504
			0.9489	0.5260	0.9498	0.9493	0.3277	0.9490	0.9500	0.2304	0.9504
			0.9506	0.5303	0.9475	0.9499	0.3288	0.9472	0.9504	0.2308	0.9492

Table IV. (Continued)

			$n = 20$			$n = 50$			$n = 100$		
			OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)	OC (13) (5) (7)	MR (13) (5) (7)	OC (16) (6) (8)
μ	σ	C_{pm}									
15.5	0.65	2.032	0.9443	1.1876	0.9523	0.9473	0.7437	0.9517	0.9489	0.5244	0.9516
			0.9439	1.1830	0.9507	0.9472	0.7426	0.9507	0.9488	0.5240	0.9510
			0.9453	1.1914	0.9481	0.9479	0.7446	0.9491	0.9492	0.5247	0.9500
15.5	0.75	1.849	0.9430	1.1070	0.9507	0.9467	0.6940	0.9509	0.9493	0.4890	0.9517
			0.9426	1.1034	0.9493	0.9468	0.6931	0.9501	0.9492	0.4887	0.9512
			0.9444	1.1116	0.9470	0.9475	0.6951	0.9484	0.9496	0.4894	0.9501
15.5	1	1.491	0.9458	0.9213	0.9487	0.9486	0.5764	0.9499	0.9482	0.4061	0.9498
			0.9455	0.9193	0.9478	0.9486	0.5759	0.9495	0.9483	0.4059	0.9495
			0.9474	0.9266	0.9452	0.9493	0.5777	0.9478	0.9486	0.4066	0.9484
15.5	1.5	1.054	0.9469	0.6635	0.9491	0.9488	0.4143	0.9495	0.9483	0.2919	0.9485
			0.9469	0.6627	0.9487	0.9489	0.4142	0.9493	0.9484	0.2918	0.9484
			0.9486	0.6681	0.9462	0.9496	0.4155	0.9475	0.9487	0.2923	0.9472
15.5	2	0.808	0.9486	0.5121	0.9495	0.9493	0.3192	0.9488	0.9493	0.2247	0.9492
			0.9485	0.5117	0.9492	0.9493	0.3192	0.9487	0.9493	0.2247	0.9492
			0.9504	0.5159	0.9466	0.9501	0.3202	0.9470	0.9497	0.2251	0.9480
16	0.15	1.648	0.9355	0.2108	0.9449	0.9447	0.1342	0.9488	0.9469	0.0951	0.9502
			0.9352	0.2107	0.9440	0.9446	0.1341	0.9484	0.9470	0.0951	0.9500
			0.9353	0.2108	0.9438	0.9447	0.1341	0.9486	0.9469	0.0951	0.9507
16	0.25	1.617	0.9370	0.3366	0.9488	0.9451	0.2136	0.9515	0.9476	0.1513	0.9498
			0.9366	0.3360	0.9473	0.9449	0.2135	0.9506	0.9476	0.1512	0.9493
			0.9368	0.3363	0.9464	0.9450	0.2136	0.9503	0.9476	0.1513	0.9493
16	0.5	1.491	0.9393	0.5577	0.9538	0.9456	0.3516	0.9530	0.9482	0.2483	0.9532
			0.9383	0.5552	0.9509	0.9452	0.3509	0.9513	0.9481	0.2481	0.9522
			0.9389	0.5569	0.9490	0.9455	0.3514	0.9502	0.9482	0.2483	0.9516
16	1	1.179	0.9432	0.6418	0.9536	0.9468	0.4026	0.9526	0.9491	0.2838	0.9532
			0.9427	0.6385	0.9510	0.9463	0.4017	0.9512	0.9487	0.2835	0.9524
			0.9439	0.6425	0.9487	0.9468	0.4027	0.9498	0.9491	0.2838	0.9515
16	1.5	0.925	0.9436	0.5534	0.9513	0.9470	0.3470	0.9499	0.9481	0.2445	0.9503
			0.9435	0.5516	0.9499	0.9469	0.3466	0.9492	0.9480	0.2443	0.9499
			0.9452	0.5557	0.9475	0.9475	0.3476	0.9475	0.9484	0.2447	0.9488
16	2	0.745	0.9456	0.4607	0.9485	0.9482	0.2881	0.9497	0.9487	0.2030	0.9511
			0.9454	0.4597	0.9477	0.9482	0.2879	0.9492	0.9488	0.2029	0.9509
			0.9473	0.4634	0.9453	0.9490	0.2888	0.9477	0.9492	0.2032	0.9498
17	0.25	0.827	0.9355	0.0885	0.9451	0.9447	0.0564	0.9480	0.9476	0.0400	0.9490
			0.9353	0.0885	0.9444	0.9446	0.0564	0.9477	0.9476	0.0400	0.9490
			0.9354	0.0885	0.9443	0.9447	0.0564	0.9482	0.9477	0.0400	0.9497

Table V. Frequencies of better coverage attainments and of coverage more proximal to the nominal by a proportion greater than 0.1% (in brackets) by the confidence intervals or lower confidence limits obtained by the new method in comparison to those obtained by Boyles's[20] methods

Estimate of δ	Confidence coefficient	Sample size	Confidence intervals (13)–(5)	Confidence intervals (13)–(7)	Lower confidence limits (16)–(6)	Lower confidence limits (16)–(8)
$\hat{\delta}_1$	90%	$n = 20$	24–0 (1–0)	7–16 (2–8)	24–0 (4–0)	24–0 (24–0)
		$n = 50$	11–2 (0–0)	11–13 (0–0)	8–6 (0–0)	22– 2 (20–0)
		$n = 100$	5–3 (0–0)	5–16 (0–0)	6–8 (0–0)	20–4 (19–2)
	95%	$n = 20$	18–0 (1–0)	9–15 (0–14)	23–1 (3–0)	24– 0 (22–0)
		$n = 50$	7–4 (0–0)	7–15 (0–0)	11–6 (0–2)	20–4 (14–3)
		$n = 100$	5–3 (0-0)	9–13 (0–0)	6–9 (0–0)	16–8 (9–3)
$\hat{\delta}_2$	90%	$n = 20$	20–0 (2–0)	8–16 (1–8)	19–3 (3–2)	24–0 (24–0)
		$n = 50$	8–3 (0–0)	5–16 (0–0)	8–7 (0–2)	21–3 (20–2)
		$n = 100$	6–2 (0–0)	10–12 (0–0)	6–9 (0–0)	20–3 (20–1)
	95%	$n = 20$	16–1 (0–0)	7–17 (1–12)	19–4 (1–4)	21–3 (20–2)
		$n = 50$	7–4 (0–0)	6–15 (0–0)	11–5 (0–2)	19–5 (16–3)
		$n = 100$	6–5 (0–0)	4–16 (0–0)	5–8 (0–0)	18–6 (11–4)

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