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Identifying the Pareto and Yule distributions by properties of their reliability measures

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Abstract

This paper provides characterizations of a family of distributions in the context of reliability theory placing emphasis on a particular family member, the Yule distribution. In particular, it is shown that the distribution of a non-negative, integer-valued random variable X with $E(X) < +\infty$ is uniquely identified to belong to the class of distributions consisting of the geometric, the Waring and the negative hypergeometric distributions if and only if anyone of the following conditions is satisfied:

- a. The mean residual life is a linear function of time.
- b. The vitality function is a linear function of time.
- c. The product of the hazard rate by the mean residual life is constant.

Characterizations of the Yule distribution based on reliability measures of its size-biased version are also provided. Continuous analogues of the results are considered. These include characterizations of the exponential and the beta of the first and second kind distributions.

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