STATISTICAL INFERENCE ON PROCESS CAPABILITY INDICES

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Abstract

Process capability indices are unitless functions of the parameters and the specifications of a process. The parameters of the process are the mean and the standard deviation and the specifications are the lower specification limit, the upper specification limit and the target value. Various indices have been proposed, but the most widely used are C_p, C_{pk}, C_{pm} and C_{pmk}. All the indices involve the parameters of the process which are usually unknown. Hence, we have to estimate the indices via a random sample from the process. Several authors have dealt with the problems of estimation and statistical inference on process capability indices when the distribution of the process is the normal distribution. The aim of the paper is to present the most widely used indices, their estimators and their drawbacks. Furthermore, some new indices are proposed that overcome the drawbacks of the standard indices and the sampling distributions of their estimators are derived via simulation.

Keywords and Phrases: process capability indices; specification limits; estimation; process yield; bootstrap method;

1. Introduction

Process capability indices (PCI's) are primarily utilized in industry in order to measure the capability of a production process to produce according to some given specifications that are related to one -or more than one- measurable characteristic (e.g. diameter, weight, length) of its produced items that can describe the process. Each of these characteristics should lie between some predetermined limits. These limits are referred to as *specification limits* or *tolerance limits*. Because of the inevitable variability, the characteristics can not take fixed values. So, they can be thought of as random variables. PCI's can be regarded, as measures of the agreement between the distributions of the characteristics and the area that is specified by the specification limits.

In the sequel, it is assumed, if not otherwise stated, that the process distribution is the normal with mean μ and standard deviation σ and that there is only one measurable characteristic. In particular, it is assumed that the process is described by a characteristic whose values must lie between two values, say L and U. The level at which the process is aimed to produce is denoted by T. The values L and U which are the minimum and the maximum allowable process values are known as the *lower* and the *upper specification limit* respectively, while the interval [L,U] is known as the *specification area*. Finally, the level T is known as the *target value*. The target value usually, coincides with the midpoint (M) of the specification area

i.e. $T=M=(L+U)/2$. This is the so called *symmetric* case. The case $T \neq M$ is called *asymmetric* case. In the sequel, both cases are examined, since the performance of the standard indices becomes sometimes unsatisfactory in the asymmetric case.

PCI's are unitless, nonnegative functions which combine the parameters of a given process (μ, σ) with its specifications (L, U, T). A large value of a PCI for a given process is normally an indication that the process is capable. Often, however, as will be seen, the use of the ordinary PCI's may lead to a high index value for an incapable process, or to an index value for a process that exceeds the corresponding index value for a more capable process.

An *ideal* process is a process whose mean coincides with the target value and its standard deviation equals zero. However, such a process is impossible to be attained in practice and the aim thus becomes to attain a process whose standard deviation is as small as possible, its mean is as close to the target value as possible and its yield is as large as possible. (The *yield* of a process is defined as the probability of producing within the specification area). For a normal process, the yield is maximized if the mean coincides with M. In the asymmetric case, where $T \neq M$, the last two aims are discordant since, keeping σ constant, we require simultaneously a mean close to the target value and a mean close to M. Hence, the location of the most desirable expected value in such situations is somewhere between M and T and depends on the magnitude of the standard deviation of the process.

The four most broadly used PCI's are C_p (Kane 1986), C_{pk} (Kane 1986), C_{pm} (Chan et al. 1988, Boyles 1991) and C_{pmk} (Pearn et al. 1992) and are defined as

$$C_p=(U-L)/6\sigma, \qquad C_{pk}=\min\{[(\mu-L)/3\sigma],[(U-\mu)/3\sigma]\},$$

$$C_{pm}=\frac{U-L}{6\sqrt{E(X-T)^2}}=\frac{U-L}{6\sqrt{\sigma^2+(\mu-T)^2}} \text{ and } C_{pmk}=\min\{\frac{\mu-L}{3\sqrt{\sigma^2+(\mu-T)^2}},\frac{U-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}\}$$

respectively.

Often, the process parameters that are involved in the computation of the indices, the mean and the standard deviation, are unknown. In such cases these parameters are usually replaced by their sample counterparts, the sample mean $(\overline{X})$ and the sample standard deviation (S) using a random sample of n observations from the examined process. Consequently, the resulting estimators of the indices C_p, C_{pk}, C_{pm} and C_{pmk} are

$$\hat{C}_p=(U-L)/6S, \quad \hat{C}_{pk}=\min\{[U-\overline{X})/3S],[(\overline{X}-L)/3S]\},$$

$$\hat{C}_{pm}=\frac{U-L}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i-T)^2}}=\frac{U-L}{6\sqrt{\frac{n-1}{n}S^2+(\overline{X}-T)^2}}$$

$$\text{and } \hat{C}_{pmk}=\min\{\frac{U-\overline{X}}{3\sqrt{\frac{n-1}{n}S^2+(\overline{X}-T)^2}},\frac{\overline{X}-L}{3\sqrt{\frac{n-1}{n}S^2+(\overline{X}-T)^2}}\},$$

respectively. Some authors have derived the exact distributions of these estimators under the assumption that the process distribution is the normal. Kotz and Johnson (1993) provide the distributions of these estimators and some approximations that have been proposed due to their complexity.

The indices C_p, C_{pk}, C_{pm} and C_{pmk}, do not perform very well in all the situations. In particular, in some situations these indices may be unable to judge the most capable among some processes, or they may judge a process as capable even if it is incapable. In particular, the index C_p ignores μ and T and assigns the same degree of capability to all the processes with common σ. Furthermore, it increases arbitrarily as σ tends to zero (even when μ is far from T). The index C_{pk} ignores T and increases arbitrarily as σ tends to zero (even for a value of μ far from that of T).

The index C_{pm} in the asymmetric case cannot distinguish between processes with the same σ whose means are equidistant from T. For example, let us assume that the specifications are L=10, U=20 and T=17.5 and that we want to compare the capabilities of processes A and B. The mean of Process A is 16 and its variance is 1, while the corresponding values for process B are 19 and 1. The index C_{pm} is 0.92 for both processes although process A is clearly more capable than B, since despite the fact that the means of the two processes are equidistant from the target value and their standard deviations are equal, the yield of process A (1) is sufficiently greater than the yield of process B (0.8413).

Finally, using C_{pmk} in the asymmetric case it is not possible to know whether the importance should be ascribed to the proximity of μ to T or of μ to M. In order to illustrate this shortcoming, let us consider a situation where L=10, U=20 and T=18 and let us have two processes. The first has mean 17 and standard deviation 0.4 and the second has mean 18.5 and standard deviation 0.4. Since both processes have common σ (0.4) and yield (1), one would expect to assess a greater C_{pmk} value for the second process while its mean is closer to T than the mean of the first. Nevertheless, C_{pmk} is 0.928 for the former and 0.781 for the latter since it is affected by the distance between μ and M despite the fact that the processes have the same yield (1). In section 2 some new indices are proposed that overcome this deficiency. Their sampling distributions are derived in section 3 using bootstrap.

2. Some new Process Capability Indices

In this section two new PCI's are proposed that perform in the asymmetric case (T≠M) better than the well known indices C_{pm} and C_{pmk}. In the symmetric case (T=M) the new indices coincide with C_{pm} and C_{pmk} respectively.

2.1 The index C'_{pm}

By its definition, the index C_{pm} measures the proximity of the process mean to the target value. In the asymmetric case, though, it is not always desirable to have processes with means as close as possible to the target value, while if the standard deviation is sufficiently large, a process with a mean different from the target value but closer to M, achieves a much better yield. Hence, if T≠M the value of μ that maximizes the index (keeping σ constant), must be intuitively somewhere between M and T if M<T, or between T and M if M>T. For simplicity, it is assumed in the sequel that M<T. However, the analysis is similar in the case M>T.

Consider now a variant of C_{pm}, denoted by C'_{pm} and defined by

$$C'_{pm} = \frac{U-L}{6\sqrt{E(X-m')^2}} = \frac{U-L}{6\sqrt{\sigma^2 + (\mu - m')^2}},$$

where $m' \in [M,T]$. This is an index which is maximized if the standard deviation is held fixed, at a value between M and T. Evidently, if M=T (symmetric case) m' =M=T and the index C'_{pm} coincides with the index C_{pm}. The value of m' should be selected so that if σ is sufficiently small, m' is proximal to T and if σ is large, m' is close to M. Finally, if σ is moderate, m' should be the midpoint of the interval [M,T]. The magnitude of variability must always be assessed from the parameter σ and from the length of the specification area [L,U].

The reason why we want the index to be maximized at M if the standard deviation goes to infinity and at T if the variance goes to zero, is that if the variability is small, the importance of having a process with a mean near the midpoint of the specification area is small too, since even a process with mean far from M may achieve a sufficiently large yield. On the other hand, if the variance of the process is very large and the mean is near to the target value but far from M, the yield of the process may be very small.

The value of m' must be a weighted average of M and T defined so that for a relatively large standard deviation the weight of M is greater than the weight of T, while for a relatively small standard deviation the weight of T is greater than the weight of M. Finally, for a moderate σ the weights of M and T are equal and so m' is the midpoint of [M,T].

In the sequel, we examine two alternative approaches for the assessment of m'. The value of m' that arises through the first approach is denoted by m'_1, while the value of m' obtained via the second is denoted by m'_2.

According to the first approach the weights of T and M are assumed to be functions of the specification limits (L and U) and of the standard deviation of the process. Thus, m'_1 can be written as $m'_1 = W_{11}(L,U,\sigma)T + W_{12}(L,U,\sigma)M$. Note that the first subscript of W_{ij} is associated with the approach and the second is associated with the weight i.e. 1 for the weight of T and 2 for the weight of M. For simplicity, we will denote $W_{11}(L,U,\sigma)$ by W_{11} and $W_{12}(L,U,\sigma)$ by W_{12}. Both W_{11} and W_{12} must take values between 0 and 1 and they sum up to unity. In addition, both W_{11} and W_{12} must be equal to 0.5 if the variability is moderate. A value of the standard deviation that can be regarded as moderate is (U-L)/1.35. The reason is that, under the assumption that μ=M, a normal process with standard deviation (U-L)/1.35 achieves a yield equal to 0.5. The weight function W_{11} must finally tend to unity if the variance tends to zero and to zero if the variance tends to infinity. Since $W_{12}=1-W_{11}$, W_{12} must go to zero if the variance tends to zero while it must go to unity if the variance tends to infinity. A function W_{11} that seems to be an appropriate selection is $W_{11}=(U-L)/(U-L+1.35\sigma)$. Indeed, this function satisfies all the required conditions since $W_{11}[L,U,\sigma=(U-L)/1.35]=0.5$, $\lim\limits_{\sigma \to 0} W_{11}(L,U,\sigma) = \lim\limits_{\sigma \to 0}[(U-L)/(U-L+1.35\sigma)]=1$ and $\lim\limits_{\sigma \to \infty} W_{11}(L,U,\sigma) = \lim\limits_{\sigma \to \infty}[(U-L)/(U-L+1.35\sigma)]=0$. Taking into account the relationship between W_{11} and W_{12} it

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follows that $W_{12}(L,U,\sigma=(U-L)/1.35)=0.5$, $\lim_{\sigma\to 0} W_{12}(L,U,\sigma)=0$ and $\lim_{\sigma\to\infty} W_{12}(L,U,\sigma)=1$. For these reasons, W_{11} and W_{12} as defined above constitute a plausible set of weights for T and M leading to

$$m_1' =[(U-L)/(U-L+1.35\sigma)]\,T+[1-(U-L)/(U-L+1.35\sigma)]M.$$

An alternative approach for assessing the value of m' would be to consider the weights of T and M to be dependent on the yield of the process. In particular, the weight of T can be defined to be equal to the process yield and the weight of M can be defined to be equal to 1-(yield). Then, since the yield is defined as $\Phi[(U-\mu)/\sigma]-\Phi[(L-\mu)/\sigma]$, we obtain $W_{21}(U,L,\mu,\sigma)=\Phi[(U-\mu)/\sigma]-\Phi[(L-\mu)/\sigma]$ and $W_{22}(U,L,\mu,\sigma)=1-\Phi[(U-\mu)/\sigma]+\Phi[(L-\mu)/\sigma]$ leading to a value of m' given by

$$m_2' =\{\Phi[(U-\mu)/\sigma]-\Phi[(L-\mu)/\sigma]\}T+\{1-\Phi[(U-\mu)/\sigma]+\Phi[(L-\mu)/\sigma]\}M.$$

Obviously, the weights in this case depend on U, L, μ and σ and so they are more general than the weights used in the assessment of m_1' which depend on L, U and σ only. Note, that if $\mu=M$ and $\sigma=(U-L)/1.35$ the values of m_1' and m_2' coincide.

Let us now examine the properties of these weights. From their definition it is apparent that $W_{21}+W_{22}=1$. In addition, $\lim_{\sigma\to 0} W_{21}(L,U,\mu,\sigma)=\Phi(\infty)-\Phi(-\infty)=1$, $\lim_{\sigma\to\infty} W_{21}(L,U,\mu,\sigma)=\Phi(0)-\Phi(0)=0$, $\lim_{\sigma\to 0} W_{22}(L,U,\mu,\sigma)=1-\Phi(\infty)+\Phi(-\infty)=0$ and $\lim_{\sigma\to\infty} W_{22}(L,U,\mu,\sigma)=1-\Phi(0)+\Phi(0)=1$. It has to be remarked that in this case a unique combination of μ and σ that gives $W_{21}=W_{22}=0.5$ does not exist, while for any given pair of specification limits, an infinite number of pairs (μ,σ) end up to processes with yield equal to 0.5.

In order to illustrate the superiority of C_{pm}' over C_{pm} in the asymmetric case, let us reconsider the example, where L=10, U=20 and T=17.5. As seen in that example, a process with $\mu=16$ and $\sigma=1$ (A) and a process with $\mu=19$ and $\sigma=1$ (B) correspond to the same value of C_{pm} (0.92), although the former is by far more capable. The use of the index C_{pm}' instead of C_{pm} overcomes this problem, since the value of C_{pm}' is 1.06 if $m'=m_1'$ and 0.92 if $m'=m_2'$ for process A and 0.81 if $m'=m_1'$ and 0.77 if $m'=m_2'$ for process B. Hence, C_{pm}' is able to detect the most capable between processes with the same standard deviation, but with different means that are equidistant from the target value. Note that for the first process $m_1'=17.2026$ and $m_2'=17.49$ and for the second process $m_1'=17.2026$ and $m_2'=17.1034$. Apparently, the value of m_1' is the same for both processes because of the equality of their standard deviations. On the other hand, the values of m_2' differ since m_2' depends also on μ and the two processes have different means.

Let us now reconsider the example in which the index C_{pmk} was unable to indicate the most capable between two processes. The index C_{pmk} resulted in the values 0.9285, for process A, and 0.7809, for process B, although as explained earlier the latter was more capable. However, the value of C_{pm}' for process A is 1.7805 or 1.5475 according as $m'=m_1'$ or

$m' = m_2'$ respectively, while the corresponding values for process B are 2.1748 or 2.6021, respectively.

The computation of the index C_{pm}' requires knowledge of the process mean and the process standard deviation. Unfortunately, these parameters are often unknown and hence it is not possible to assess the actual value of C_{pm}' for a given process. Instead of the index C_{pm}' one may obtain an estimate of it, based on a random sample $X_1,...,X_n$ from the examined process. A plausible estimator of C_{pm}' denoted by $\hat{C}_{pm}'$, is

$$\hat{C}_{pm}' = \frac{U-L}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \hat{m}')^2}} = \frac{U-L}{6\sqrt{\frac{n-1}{n}S^2 + (\overline{X} - \hat{m}')^2}},$$

where $\hat{m}'$ can be any of $\hat{m}_1' = [(U-L)/(U-L+1.35S)]T + [1-(U-L)/(U-L+1.35\sigma)]M$ or $\hat{m}_2' = \{\Phi[(U-\overline{X})/S] - \Phi[(L-\overline{X})/S]\}T + \{1-\Phi[(U-\overline{X})/S] + \Phi[(L-\overline{X})/S]\}M$.

If M=T, $\hat{m}' = M = T$ and the estimator $\hat{C}_{pm}'$ coincides with the estimator $\hat{C}_{pm}$ of the index C_{pm} that was proposed by Boyles (1991). This estimator can be used for making statistical inference for C_{pm}, and so if T=M for C_{pm}'. Boyles (1991) suggests two approximations of the distribution of $\hat{C}_{pm}$.

2.2 The index C_{pmk}'

The value m' defined above can also be used in connection with the index C_{pmk} leading to a new index, denoted by C_{pmk}', thus

$$C_{pmk}' = \min\{\frac{\mu - L'}{3\sqrt{E(X-m')^2}}, \frac{U'-\mu}{3\sqrt{E(X-m')^2}}\} = \min\{\frac{\mu - L'}{3\sqrt{\sigma^2 + (\mu - m')^2}}, \frac{U'-\mu}{3\sqrt{\sigma^2 + (\mu - m')^2}}\},$$

where m' can be any of m_1' or m_2' defined earlier, $L' = m'-d$, $U' = m'+d$ and $d = \min\{m'-L, U-m'\}$. Note that, C_{pmk}' is set to be equal to zero if found to be negative. Evidently, if T=M, then $m' = T = M = (L+U)/2$ and so $d = (U-L)/2$, $L' = M-(U-L)/2 = L$ and $U' = M+(U-L)/2 = U$. Consequently, in the symmetric case the index C_{pmk}' coincides with the index C_{pmk}.

In order to illustrate the superiority of C_{pmk}' over C_{pmk}, let us now reconsider the fifth example of Section 3 where the index C_{pmk} does not perform well. According to C_{pmk}, the first process is judged more capable although both processes achieve yields equal to unity and the mean of the second process is closer to T. The value of C_{pmk} is 0.9285 for the first process and 0.7809 for the second. The index C_{pmk}' results in the values 0.46 if $m' = m_1'$ and 0.30 if $m' = m_2'$ for the first process and the values 0.65 and 0.78 if $m' = m_1'$ and $m' = m_2'$ respectively for the second process. Hence, through the use of this index the superiority of the second process is revealed. Note that $m_1' = 17.84$ and $m_2' = 18$ for the first process, while the corresponding values for the second process are $m_1' = 17.84$ and $m_2' = 17.99$ respectively.

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A plausible estimator of the index C'_{pmk} is

$$\hat{C}'_{pmk} = \min\left\{ \frac{\overline{X} - \hat{L}'}{3\sqrt{\dfrac{n-1}{n}S^2 + \left(\overline{X} - \hat{m}'\right)^2}}, \frac{\hat{U}' - \overline{X}}{3\sqrt{\dfrac{n-1}{n}S^2 + \left(\overline{X} - \hat{m}'\right)^2}} \right\},$$

where $\hat{m}'$ can be any of $\hat{m}'_1$ or $\hat{m}'_2$ that were defined above, $\hat{L}' = \hat{m}' - \hat{d}$, $\hat{U}' = \hat{m}' + \hat{d}$, and finally $\hat{d} = \min\{\hat{m}' - L, U - \hat{m}'\}$. The distribution of the estimator $\hat{C}'_{pmk}$ is fairly intractable. However, if T=M, the estimator $\hat{C}'_{pmk}$ coincides with the estimator of C_{pmk} that Pearn et al. (1992) proposed. This estimator is denoted by $\hat{C}_{pmk}$ and its sampling distribution is

$$\left(\frac{d}{v} - \chi'_1(\lambda)\right) \Big/ \left(3\sqrt{\chi^2_{n-1} + \chi'^2_1(\lambda)}\right)$$

where d=(U-L)/2, $v = \sigma/\sqrt{n}$ and $\chi'^2_1(\lambda)$ is the noncentral chi-square distribution with 1 degree of freedom and noncentrality parameter $\lambda = n(\mu - T)^2/\sigma^2$.

3. The Sampling Distributions of the Index Estimators via the Bootstrap Method – A Simulation Study

In this section, the sampling distributions of the estimators of the indices introduced above are derived via the bootstrap method of estimation (Efron and Tibshirani, 1993). In particular, a simulation study is performed so as to examine the forms of the sampling distributions of the estimators $\hat{C}_p$, $\hat{C}_{pk}$, $\hat{C}_{pm}$, $\hat{C}_{pmk}$, $\hat{C}'_{pm}$ (with m' equal to m'_1 or m'_2) and $\hat{C}'_{pmk}$ (with m' equal to m'_1 or m'_2) and how the behave as the sample size (n) increases.

Under the assumption that the specifications are L=10, U=20 and T=16 random samples of size 20, 100 and 200 were generated from the normal distribution with parameters μ=14, 16 and 17 and σ=1 and 2. For all these combinations, bootstrap samples of size B=2000 were generated from the initial samples and the values of the estimators $\hat{C}_p$, $\hat{C}_{pk}$, $\hat{C}_{pm}$, $\hat{C}_{pmk}$, $\hat{C}'_{pm1}$ (i.e. $\hat{C}'_{pm}$ with $m' = m'_1$), $\hat{C}'_{pm2}$ (i.e. $\hat{C}'_{pm}$ with $m' = m'_2$), $\hat{C}'_{pmk1}$ (i.e. $\hat{C}'_{pmk}$ with $m' = m'_1$), and $\hat{C}'_{pmk2}$ (i.e. $\hat{C}'_{pmk}$ with $m' = m'_2$) were assessed, for each of the 2000 samples.

Since, the bootstrap distribution of an estimator is an estimate of its sampling distribution, the B bootstrap values of each estimator were used to check if its distribution resembles the normal. Tables 8.1, 8.2 and 8.3 provide the p-values of the Kolmogorov – Smirnov (KS) test for n=20, n=100 and n=200 respectively, for all the combinations of the values of μ and σ considered. Recall that the null hypothesis of the KS test is normality while the alternative is nonnormality. It can be observed that when n=20 (Table 8.1), the sampling distributions are far from being normal. (Only two of the p-values are greater than 0.05). However, as n increases the sampling distributions of all the estimators tend to normality (table 8.2 and 8.3).

Table 8.1 KS test p-values for n=20

Index	Distribution	C_p	C_{pk}	C_{pm}	C_{pmk}	C_{pm1}	C_{pm2}	C_{pmk1}	C_{pmk2}
p-value	$N(14,1^2)$	.00	.00	.007	.004	.004	.006	0.004	.007
p-value	$N(16,1^2)$	.00	.00	.00	.00	.00	.00	0.003	.00
p-value	$N(17,1^2)$	.00	.00	.00	.003	.00	.00	0.003	.004
p-value	$N(14,2^2)$	.00	.00	.00	.00	.00	.00	0.001	.00
p-value	$N(16,2^2)$	.00	.00	.00	.011	.00	.00	**0.343**	**.385**
p-value	$N(17,2^2)$	.00	.00	.001	.002	.00	.001	0.001	.001

Table 8.2 KS test p-values for n=100

Index	Distribution	C_p	C_{pk}	C_{pm}	C_{pmk}	C_{pm1}	C_{pm2}	C_{pmk1}	C_{pmk2}
p-value	$N(14,1^2)$	**.344**	.047	**.057**	**.105**	**.065**	**.054**	**.091**	**.063**
p-value	$N(16,1^2)$	.024	**.067**	.024	**.088**	**.051**	.026	**.354**	**.059**
p-value	$N(17,1^2)$	**.096**	.013	.022	.031	.024	.023	.038	.036
p-value	$N(14,2^2)$	.00	.042	**.061**	.016	**.076**	.047	.004	.004
p-value	$N(16,2^2)$	**.116**	**.100**	**.095**	**.347**	.030	**.104**	**.517**	**.219**
p-value	$N(17,2^2)$	**.201**	**.376**	**.182**	**.229**	**.130**	**.217**	**.115**	**.238**

Table 8.3 KS test p-values for n=200

Index	Distribution	C_p	C_{pk}	C_{pm}	C_{pmk}	C_{pm1}	C_{pm2}	C_{pmk1}	C_{pmk2}
p-value	$N(14,1^2)$	**.105**	**.166**	**.146**	**.255**	**.167**	**.148**	**.375**	**.290**
p-value	$N(16,1^2)$	.038	.037	.023	**.052**	**.050**	.024	**.109**	**.052**
p-value	$N(17,1^2)$	**.231**	**.668**	**.195**	**.299**	**.184**	**.208**	**.306**	**.287**
p-value	$N(14,2^2)$	**.297**	**.723**	**.485**	**.208**	**.472**	**.425**	**.426**	**.315**
p-value	$N(16,2^2)$	.006	**.428**	.025	**.537**	.023	.019	**.489**	**.810**
p-value	$N(17,2^2)$	**.237**	**.187**	**.176**	**.191**	**.118**	**.184**	**.125**	**.196**

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