



## Evaluating volatility forecasts in option pricing in the context of a simulated options market

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### Abstract

The performance of an ARCH model selection algorithm based on the standardized prediction error criterion (SPEC) is evaluated. The evaluation of the algorithm is performed by comparing different volatility forecasts in option pricing through the simulation of an options market. Traders employing the SPEC model selection algorithm use the model with the lowest sum of squared standardized one-step-ahead prediction errors for obtaining their volatility forecast. The cumulative profits of the participants in pricing 1-day index straddle options always using variance forecasts obtained by GARCH, EGARCH and TARARCH models are compared to those made by the participants using variance forecasts obtained by models suggested by the SPEC algorithm. The straddles are priced on the Standard and Poor 500 (S & P 500) index. It is concluded that traders, who base their selection of an ARCH model on the SPEC algorithm, achieve higher profits than those, who use only a single ARCH model. Moreover, the SPEC algorithm is compared with other criteria of model selection that measure the ability of the ARCH models to forecast the realized intra-day volatility. In this case too, the SPEC algorithm users achieve the highest returns. Thus, the SPEC model selection method appears to be a useful tool in selecting the appropriate model for estimating future volatility in pricing derivatives.

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## 1. Introduction

ARCH models have widely been used in financial time series analysis and particularly in analyzing the risk of holding an asset, evaluating the price of an option, forecasting time varying confidence intervals and obtaining more efficient estimators under the existence of heteroscedasticity. In the recent literature, numerous parametric specifications of ARCH models have been considered for the description of the characteristics of financial markets. The richness of the family of parametric ARCH models certainly complicates the search for the true model, and leaves quite a bit of arbitrariness in the model selection stage. The problem of selecting the model that describes best the movement of the series under study is therefore of practical importance.

Degiannakis and Xekalaki (1999) made a comparative study of the forecasting ability of ARCH models based on a standardized prediction error criterion (SPEC). According to their SPEC-based model selection algorithm, the models with the lowest sum of squared standardized one-step-ahead prediction errors are the most appropriate to exploit future volatility. In this paper, inspired by Engle et al.'s (1993) approach to assess incremental profits for a set of competing forecasts of the variance for a given portfolio, we examine the usage of the SPEC model selection algorithm, in pricing contingent claims. The goal of the paper is to evaluate the SPEC algorithm for volatility model selection through the simulation of an options market. In particular, in Section 2, the ARCH process is presented. Section 3 provides a description of the SPEC model selection algorithm in the context of ARCH models, while Section 4 presents Engle et al.'s (1993) data generated set-up of evaluating volatility forecasts. In Sections 5 and 6, based on Engle et al.'s (1993) technique, the suggested model selection method is evaluated using daily return data for the S&P500 stock index over the period from June 26th, 1991 to October 18th, 2002. The use of a model selection method is a tedious procedure as it presupposes the estimation of a set of models. In order to examine whether there is any added value in using the suggested model selection algorithm instead of any other method of using only a single ARCH model in the study, the performance of the SPEC algorithm is investigated against a set of such methods for a range of ARCH models. The results of Section 5 provide evidence that this is indeed the case since they indicate that the SPEC model selection algorithm offers a useful tool in providing information related to the appropriate model. In Section 6, the algorithm is compared with other methods of model selection. In particular, model selection criteria that measure the accuracy of the models to predict the realized volatility are constructed. The SPEC method is then compared with those model selection methods. Clearly, the SPEC algorithm outperforms all of the other methods of model selection considered. Finally, in Section 7 a brief discussion of the results is provided.

## 2. The autoregressive conditional heteroscedasticity (ARCH) process

Let  $\{y_t\}$  refer to the univariate discrete time real-valued stochastic process to be predicted (i.e. the rate of return of a particular stock or market portfolio from time  $t - 1$  to  $t$ ), where  $E(y_t|I_{t-1}) \equiv E_{t-1}(y_t) \equiv \mu_t$  denotes the conditional mean given the information set available at time  $t - 1$ ,  $I_{t-1}$ . The innovation process for the conditional mean,  $\{\varepsilon_t\}$ , is then given by

$\varepsilon_t = y_t - \mu_t$  with zero unconditional mean, unconditional variance  $V(\varepsilon_t) = E(\varepsilon_t^2) \equiv \sigma^2$  and  $E(\varepsilon_t \varepsilon_s) = 0, \forall t \neq s$ . The conditional variance of the process is allowed to depend on  $I_{t-1}$  and is given by  $V_{t-1}(y_t) = E_{t-1}(\varepsilon_t^2) \equiv \sigma_t^2$ . Since investors would know the information set  $I_{t-1}$  when they make their investment decisions at time  $t - 1$ , the relevant expected return to the investors and volatility are  $\mu_t$  and  $\sigma_t^2$ , respectively.

An ARCH process,  $\{\varepsilon_t\}$ , can be presented as

$$\varepsilon_t = z_t \sigma_t,$$

where

$$z_t \stackrel{\text{i.i.d.}}{\sim} f[E(z_t) = 0, V(z_t) = 1] \tag{1}$$

and

$$\sigma_t^2 = g(\sigma_{t-1}, \sigma_{t-2}, \dots; \varepsilon_{t-1}, \varepsilon_{t-2}, \dots; v_{t-1}, v_{t-2}, \dots),$$

where  $\{z_t\}$  is a sequence of independently and identically distributed random variables,  $f(\cdot)$  is the density function of  $z_t$ ,  $\sigma_t$  is a time-varying, positive and measurable function of the information set at time  $t - 1$ ,  $v_t$  is a vector of predetermined variables included in  $I_t$  and  $g(\cdot)$  could be a linear or nonlinear functional form that has been considered in the ARCH literature. By definition,  $\varepsilon_t$  is serially uncorrelated with mean zero, but with a time varying conditional variance equal to  $\sigma_t^2$ . The conditional variance is a linear or nonlinear function of lagged values of  $\sigma_t$  and  $\varepsilon_t$  and predetermined variables included in  $I_{t-1}$ ,  $(v_{t-1}, v_{t-2}, \dots)$ . The standard ARCH models assume that  $f(\cdot)$  is the density function of the normal distribution. However, [Bollerslev \(1987\)](#) proposed using the student  $t$  distribution with an estimated kurtosis regulated by the degrees of freedom parameter. [Nelson \(1991\)](#) proposed using the generalized error distribution ([Harvey, 1981](#); [Box and Tiao, 1973](#)), which is also referred to as the exponential power distribution. Other distributions, that have been employed, include the generalized  $t$  distribution ([Bollerslev et al., 1994](#)), the skewed student  $t$  distribution ([Lambert and Laurent, 2000, 2001](#)), the normal Poisson mixture distribution ([Jorion, 1988](#)), the normal lognormal mixture distribution ([Hsieh, 1989](#)) and a mixture of the distributions of normal variables ([Cai, 1994](#)) or student  $t$  variables ([Hamilton and Susmel, 1994](#)). Since very few financial time series have a constant conditional mean of zero, an ARCH model can be presented in a regression form by letting  $\varepsilon_t$  be the innovation process in a linear regression

$$y_t = x_t' \beta + \varepsilon_t,$$

where

$$\varepsilon_t | I_{t-1} \sim f(0, \sigma_t^2)$$

and

$$\sigma_t^2 = g(\sigma_{t-1}, \sigma_{t-2}, \dots; \varepsilon_{t-1}, \varepsilon_{t-2}, \dots; v_{t-1}, v_{t-2}, \dots),$$

where  $x_t$  is a  $k \times 1$  vector of endogenous and exogenous explanatory variables included in the information set  $I_{t-1}$  and  $\beta$  is a  $k \times 1$  vector of unknown parameters.

Engle (1982) introduced the original form of  $\sigma_t^2 = g(\cdot)$  as a linear function of the past  $q$  squared innovations

$$\sigma_t^2 = a_0 + \sum_{i=1}^q (a_i \varepsilon_{t-i}^2).$$

For the conditional variance to be positive, the parameters must satisfy  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$ , for  $i = 1, \dots, q$ . Bollerslev (1986) proposed the generalized ARCH, or GARCH( $p, q$ ), model, where the conditional variance is postulated to be a linear function of both the past  $q$  squared innovations and the past  $p$  conditional variances

$$\sigma_t^2 = a_0 + \sum_{i=1}^q (a_i \varepsilon_{t-i}^2) + \sum_{j=1}^p (b_j \sigma_{t-j}^2), \quad (2)$$

where  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$ , for  $i = 1, \dots, q$ , and  $b_j \geq 0$ , for  $j = 1, \dots, p$ . Note that even though the innovation process for the conditional mean is serially uncorrelated, it is not independent through time. The innovations for the variance are denoted as

$$E_t(\varepsilon_t^2) - E_{t-1}(\varepsilon_t^2) = \varepsilon_t^2 - \sigma_t^2 \equiv v_t.$$

The innovation process  $\{v_t\}$  is a martingale difference sequence in the sense that it cannot be predicted from its past. However, its range may depend upon the past, making it neither serially independent nor identically distributed.

The unconditional distribution of  $\varepsilon_t$  has fatter tails than the time invariant distribution of  $z_t$ . For example, for the ARCH process in (1) with density function  $f(\cdot)$  being that of the normal distribution and with the functional form of  $\sigma_t^2$  denoted as in the ARCH(1) model, the kurtosis of  $\varepsilon_t$  is  $E(\varepsilon_t^4)/E(\varepsilon_t^2)^2 = 3(1 - \alpha_1^2)/(1 - 3\alpha_1^2)$ . This is always greater than 3, the kurtosis value of the normal distribution.

The GARCH( $p, q$ ) model successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering first noted by Mandelbrot (1963). On the other hand, the GARCH structure imposes important limitations. The variance only depends on the magnitude and not the sign of  $\varepsilon_t$ , which is somewhat at odds with the empirical behaviour of stock market prices where a leverage effect may be present. The leverage effect, first noted by Black (1976), refers to the tendency for changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to bad news, ( $\varepsilon_t < 0$ ), and to fall in response to good news, ( $\varepsilon_t > 0$ ).

In order to capture the asymmetry exhibited by the data, a new class of models was introduced, the asymmetric ARCH models. The most popular model proposed to capture the asymmetric effects is Nelson's (1991) exponential GARCH, or EGARCH( $p, q$ ), model

$$\ln(\sigma_t^2) = a_0 + \sum_{i=1}^q \left( a_i \left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - E \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^p (b_j \ln(\sigma_{t-j}^2)). \quad (3)$$

The parameter  $\gamma_1$  allows for the asymmetric effect. If  $\gamma_1 = 0$ , a positive surprise, ( $\varepsilon_{t-1} > 0$ ), has the same effect on volatility as a negative surprise, ( $\varepsilon_{t-1} < 0$ ). Here, the term surprise at time  $t$  refers to the unexpected return, which is the rate of return from time  $t - 1$  to  $t$

minus the relevant expected return to the investors, e.g.,  $\varepsilon_t = y_t - \mu_t$ . If  $-1 < \gamma_1 < 0$ , a positive surprise increases volatility less than a negative surprise. If  $\gamma_1 < -1$ , a positive surprise actually reduces volatility while a negative surprise increases volatility. For  $\gamma_1 < 0$ , the leverage effect exists. Since (3) is an expression of  $\ln \sigma_t^2$ , the conditional variance,  $\sigma_t^2$ , is guaranteed to be positive. Thus, in contrast to the GARCH model, no restrictions need to be imposed on the model estimation.

The threshold GARCH, or TARCH( $p, q$ ), model is another widely used asymmetric model

$$\sigma_t^2 = a_0 + \sum_{i=1}^q (a_i \varepsilon_{t-i}^2) + \gamma \varepsilon_{t-1}^2 D(\varepsilon_{t-1} < 0) + \sum_{j=1}^p (b_j \sigma_{t-j}^2), \tag{4}$$

where  $D(\varepsilon_t < 0) = 1$  if  $\varepsilon_t < 0$ , and  $D(\varepsilon_t < 0) = 0$  otherwise. The model was introduced independently by [Zakoian \(1990\)](#) and [Glosten et al. \(1993\)](#). The TARCH model allows a response of volatility to news with different coefficients for good and bad news. Therefore, good news has an impact of  $\sum_{i=1}^q a_i$ , while bad news has an impact of  $\sum_{i=1}^q (a_i) + \gamma$ .

Other popular asymmetric models are the asymmetric power ARCH, or APARCH, model introduced by [Ding et al. \(1993\)](#), the nonlinear asymmetric GARCH, or NAGARCH, model introduced by [Engle and Ng \(1993\)](#) and the quadratic ARCH, or QGARCH, model, introduced by [Sentana \(1995\)](#). ARCH models go by such exotic names as AARCH, NARCH, PARCH, PNP-ARCH and STARARCH among others. A wide range of proposed ARCH models is covered in surveys such as [Bollerslev et al. \(1992\)](#), [Bollerslev et al. \(1994\)](#), [Bera and Higgins \(1993\)](#), [Gourieroux \(1997\)](#) and [Degiannakis and Xekalaki \(2004\)](#).

### 3. The standardized prediction error criterion (SPEC) model selection method

Let us consider the set of  $M$  candidate ARCH models of the form

$$y_t = x_t^{(m)} \beta^{(m)} + \varepsilon_t^{(m)},$$

where

$$\varepsilon_t^{(m)} = z_{1,t} \sigma_t^{(m)},$$

$$z_{1,t} \stackrel{\text{i.i.d}}{\sim} N(0, 1)$$

and

$$\sigma_t^{2(m)} = g(\sigma_{t-1}^{2(m)}, \dots, \sigma_{t-p}^{2(m)}, \varepsilon_{t-1}^{2(m)}, \dots, \varepsilon_{t-q}^{2(m)}, v_{t-1}^{(m)}, v_{t-2}^{(m)}, \dots),$$

where the superscript  $m$  refers to model  $m$ ,  $m = 1, 2, \dots, M$ . Assume that, at each of a series of points in time, we are interested in looking for the most suitable of the  $M$  competing models for obtaining a volatility forecast. [Degiannakis and Xekalaki \(1999\)](#), in the context of comparatively evaluating the predictive ability of ARCH models considered using an algorithm based on a multi-model variant of [Xekalaki et al. \(2003\)](#) two-model procedure that utilizes a standardized prediction error criterion (SPEC). According to the SPEC model

Table 1  
The estimation steps required at time  $k$  for each model  $m$  by the SPEC model selection algorithm

Model	Time				
	$k = T$	$k = T + 1$	. . .	$k = T + j$	. . .
$m = 1$	$\sum_{t=1}^T \hat{r}_t^{2(1)}$	$\sum_{t=2}^{T+1} \hat{r}_t^{2(1)}$	. . .	$\sum_{t=j+1}^{T+j} \hat{r}_t^{2(1)}$	. . .
$m = 2$	$\sum_{t=1}^T \hat{r}_t^{2(2)}$	$\sum_{t=2}^{T+1} \hat{r}_t^{2(2)}$	. . .	$\sum_{t=j+1}^{T+j} \hat{r}_t^{2(2)}$	. . .
.	.	.	.	.	.
$m = i$	$\sum_{t=1}^T \hat{r}_t^{2(i)}$	$\sum_{t=2}^{T+1} \hat{r}_t^{2(i)}$	. . .	$\sum_{t=j+1}^{T+j} \hat{r}_t^{2(i)}$	. . .
.	.	.	.	.	.
$m = M$	$\sum_{t=1}^T \hat{r}_t^{2(M)}$	$\sum_{t=2}^{T+1} \hat{r}_t^{2(M)}$	. . .	$\sum_{t=j+1}^{T+j} \hat{r}_t^{2(M)}$	. . .

At time  $k$  ( $k = T, T + 1, \dots$ ), select the model  $m$  with the minimum value for the sum of the squares of the  $T$  most recent standardized one-step-ahead prediction errors,  $\sum_{t=k-T+1}^k \hat{r}_t^{2(m)} \equiv \sum_{t=k-T+1}^k \hat{\varepsilon}_{t|t-1}^{2(m)} / \hat{\sigma}_{t|t-1}^{2(m)}$ .

selection algorithm, the model with the lowest sum of squared standardized one-step-ahead prediction errors is considered as having a better ability to predict the conditional variance of the dependent variable. Thus, at time  $k$ , selecting a strategy for the most appropriate model to forecast volatility at time  $k + 1$  ( $k = T, T + 1, \dots$ ) could naturally amount to selecting the model, which, at time  $k$ , has the lowest value of standardized one-step-ahead prediction errors,  $\sum_{t=k-T+1}^k \hat{r}_t^{2(m)} \equiv \sum_{t=k-T+1}^k \hat{\varepsilon}_{t|t-1}^{2(m)} / \hat{\sigma}_{t|t-1}^{2(m)}$ . Here,  $\hat{\varepsilon}_{t|t-1}^{(m)} = y_t - x_t^{(m)} \hat{\beta}_{t-1}^{(m)}$  is the one-step-ahead prediction error of model  $m$ , where  $\hat{\beta}_{t-1}^{(m)}$  is the estimator of  $\beta^{(m)}$  based on the information set that is available at time  $t - 1$  and  $\hat{\sigma}_{t|t-1}^{2(m)}$  is the one-step-ahead conditional variance forecast of model  $m$ . The estimation steps comprising the SPEC model selection algorithm are summarized in Table 1.

The rows of this table refer to candidate ARCH models, the columns refer to trading days, while its entries represent the sums of the squares of the  $T$  most recent standardized one-step-ahead prediction errors of each of the  $M$  models. Each trading day, the choice of the model to be used to predict the conditional variance for the next day is determined by the entry of the corresponding column of Table 1 that has the minimum value. In particular, model  $m = i$  will be chosen at time  $k = T + j$  if it is the one that corresponds to the cell of column  $T + j$  that has the minimum value of  $\sum_{t=j+1}^{T+j} \hat{r}_t^{2(i)}$ .

#### 4. Evaluation of variance forecasts with simulated option prices

As Engle et al. (1997, p. 120) noted, “a natural criterion for choosing between any pair of competing methods to forecast the variance of the rate of return on an asset would be the

expected incremental profit from replacing the lesser forecast with the better one". Engle et al. (1993) considered evaluating variance forecasts of the NYSE index using generated index option prices instead of actual ones, thus avoiding the perennial problems inherent in observed option prices. The wildcard delivery option on cash-settled options (the right of an option buyer to exercise up an option at the closing price for a period of time after the close of stock market), the existence of bid–ask spread and transaction costs, the non-synchronous coexistence of option and stock prices, are some of the difficulties that are induced in empirical studies by the use of the actual index–option prices. In particular, Engle et al. (1993) used a set of competing methods to generate alternative daily forecasts for the variance of the returns on the NYSE index and applied these forecasts to price 1-day options on \$1 shares of the NYSE index. The moving average variance, the ordinary least squares, the ARMA(1,1) in the squared residuals and the GARCH(1,1) models were applied for 3 sample lengths of (i) 300 days, (ii) 1000 days, and (iii) 5000 days. The four models and the three sample lengths produce 12 variance forecasts predicting methods. To these, Engle et al. (1993) added 3 more predicting methods by considering the average of all daily forecasts, the daily minimum and the daily maximum forecasts. As reported by Kane and Marks (1987), the average of conditionally independent forecasts converges rapidly to a perfect forecast, so that any failure of the average forecast might be indicative of departures from quality and conditional independence of the individual forecasts. As a check for the presence of bias, Engle et al. (1993) added the minimum and maximum of the daily forecasts. So, for example, in case of a significant downward bias, the maximum forecast will beat the minimum forecast and all of the individual forecasts that are more severe biased.

The pricing of index options is based on the Black and Scholes option pricing formula (Black and Scholes, 1973). In particular, the forecast price of a call and a put option at time  $t + 1$  given the information available at time  $t$ , with  $\tau$  days to maturity, denoted, respectively, by  $C_{t+1|t}^{(\tau)}$  and  $P_{t+1|t}^{(\tau)}$  are given by

$$C_{t+1|t}^{(\tau)} = S_t N(d_1) - K e^{-rf_t \tau} N(d_2),$$

$$P_{t+1|t}^{(\tau)} = -S_t N(-d_1) + K e^{-rf_t \tau} N(-d_2),$$

where

$$d_1 = \frac{\ln(S_t/K) + (rf_t + 1/2(\sigma_{t+1|t}^{(\tau)})^2)\tau}{\sigma_{t+1|t}^{(\tau)}\sqrt{\tau}} \quad \text{and} \quad d_2 = d_1 - \sigma_{t+1|t}^{(\tau)}\sqrt{\tau}.$$

Here,  $S_t$  is the market closing price of the stock (or portfolio) at time  $t$  (used as a forecast for  $S_{t+1}$ ),  $rf_t$  is the daily continuously compounded risk free interest rate and  $K$  is the exercise (or strike) price at maturity day, while,  $N(\cdot)$  and  $\sigma_{t+1|t}^{(\tau)}$  denote, respectively, the cumulative normal distribution function and the standard deviation of the rate of return during the life of the option, from  $t + 1$  until the maturity day, given the information available at time  $t$ .

Each agent applies a variance-forecast method and trades one-day options on a \$1 share of the NYSE portfolio. The exercise price is taken to be  $\exp(rf_t)$ . Thus, for  $S_t = 1$ ,  $\tau = 1$ ,

$K = \exp(rf_t)$ ,  $\sigma_{t+1|t}^{(1)} \equiv \hat{\sigma}_{t+1|t}$ ,  $C_{t+1|t}^{(1)} \equiv C_{t+1|t}$  and  $P_{t+1|t}^{(1)} \equiv P_{t+1|t}$ , the Black and Scholes option pricing formula reduces to

$$C_{t+1|t} = P_{t+1|t} = 2N(0.5\hat{\sigma}_{t+1|t}) - 1.$$

The way in which the simulated options market operates is the following: the daily differences in the variance forecasts of the various methods considered lead to different reservation prices for 1-day options on the index considered. These, in turn, trigger option trading among fictitious agents, each using one of the forecast methods considered. A trader with a higher (or lower) variance forecast and, hence, with a higher (or lower) reservation price for the option would buy (or sell) a straddle on a \$1 share of index considered from any of the remaining traders with lower (or higher) reservation prices for the option. A straddle option is the purchase (or sale) of both a call and a put option, of the underlying asset, with the same maturity day. The straddle trading is used because a straddle, that has its stock price equal to the exercise price, is Delta neutral. Delta is the change in the option price for a given change in the stock price

$$\Delta_{CALL} = \frac{\partial C}{\partial S} = e^{-\gamma\tau}N(d_1) > 0$$

and

$$\Delta_{PUT} = \frac{\partial P}{\partial S} = e^{-\gamma\tau}(N(d_1) - 1) < 0.$$

The day  $t$  payoff to agent  $i$  from holding the straddle is

$$\pi_t = \max(\exp(y_t) - \exp(r_t), \exp(r_t) - \exp(y_t)),$$

which is identical for each agent. A trade between two agents,  $i$  and  $i^*$ , is executed at the average of the reservation prices of the two agents, that is, at the bid/ask prices. The transaction that is executed at the average of the bid and ask price, yields to agent  $i$  a profit given by

$$\pi_{t+1}^{(i,i^*)} = \begin{cases} \pi_{t+1} - (C_{t+1|t,(i)} + C_{t+1|t,(i^*)}), & \text{for } C_{t+1|t,(i)} > C_{t+1|t,(i^*)}, \\ (C_{t+1|t,(i)} + C_{t+1|t,(i^*)}) - \pi_{t+1}, & \text{for } C_{t+1|t,(i)} < C_{t+1|t,(i^*)}. \end{cases}$$

In Engle et al. (1993), the GARCH(1,1) forecast method achieves the highest cumulative profits for the three sample lengths. Moreover, the GARCH(1,1) method for a rolling sample of 1000 observations yields the highest profit, dominating even the average of all variance forecast methods.

### 5. Evaluating the SPEC model selection algorithm on simulated options

Degiannakis and Xekalaki (2001b) applied a number of statistical evaluation criteria in order to examine the ability of the SPEC model selection algorithm to select the ARCH model that best predicts future volatility, for forecast horizons ranging from one day ahead to one hundred days ahead. Their results showed that the SPEC model selection procedure

has a satisfactory performance in selecting that model that generates “better” volatility predictions. Moreover, Degiannakis and Xekalaki (2001a) made a comparative study among a set of ARCH model selection algorithms in order to examine which method yields the highest profits in straddle trading based on volatility forecasts using actual option price data. The results showed that the SPEC algorithm for  $T = 5$  achieved the highest rate of return.

In the sequel, the performance of the SPEC algorithm as an ARCH model selection criterion is evaluated in the context of a simulated options market in order to avoid biases induced by the use of actual index-option prices. In particular, following Engle et al.’s (1993) approach, an economic criterion to evaluate the SPEC model selection algorithm is adopted: the profit from variance forecasts in pricing 1-day index straddle options. A simulated market of option trading among 104 fictitious agents is created, whereby traders use variance forecasts obtained by the models of their choice to price a straddle on the S&P500 index for the next day. The performance of the SPEC algorithm is evaluated through comparing the different volatility forecasts. The comparison is performed on the basis of the cumulative profits of traders each of which always uses volatility forecasts obtained by the same GARCH, EGARCH or TARARCH model on the one hand and cumulative profits by traders using volatility forecasts obtained by models suggested by the SPEC criterion on the other. So, traders can be thought of a having different “methods” or “strategies” for obtaining variance forecasts (amounting to the utilization of the forecasts of a model at each point in time) and can be classified into two categories: those who choose to always use one and the same ARCH model and those who at each point in time choose to use the ARCH model suggested by the SPEC algorithm. The variance forecast methods that are compared are: 85 selection “methods” (strategies), one for each of 85 ARCH models, each amounting to the utilization of the forecasts of the same model at any point in time, the SPEC model selection algorithm for 16 different sample sizes, the average, the minimum and the maximum of all daily forecasts methods.

The data set consists of S&P500 stock index daily returns in the period from June 26th, 1991 to October 18th, 2002, totally 2853 trading days.

The conditional mean is considered as a  $\kappa$ th order autoregressive process

$$y_t = \mu_t + z_t \sigma_t,$$

$$\mu_t = c_0 + \sum_{i=1}^{\kappa} (c_i y_{t-i})$$

and

$$z_t \stackrel{\text{i.i.d.}}{\sim} N(0, 1).$$

Usually, the conditional mean is either the overall mean or a first-order autoregressive process. Theoretically, the  $AR(1)$  process allows for the autocorrelation induced by discontinuous (or non-synchronous) trading in the stocks making up an index (Scholes and Williams, 1977, Lo and MacKinlay, 1988). According to Campbell et al. (1997), “the non-synchronous trading arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other,

possible irregular lengths”. Higher orders of the autoregressive process are considered in order to investigate if they are adequate to produce more accurate predictions.

The conditional variance is regarded as a GARCH( $p, q$ ), an EGARCH( $p, q$ ) and a TARCH( $p, q$ ) function in the forms of (2)–(4), respectively. Thus, the AR( $\kappa$ )GARCH( $p, q$ ), AR( $\kappa$ )EGARCH( $p, q$ ) and AR( $\kappa$ )TARCH( $p, q$ ) models are applied, for  $\kappa = 0, \dots, 4$ ,  $p = 0, 1, 2$  and  $q = 1, 2$ , yielding a total of 85 cases. Numerical maximization of the log-likelihood function, for the E-GARCH(2,2) model, frequently failed to converge. So the five E-GARCH models for  $p = q = 2$  were excluded. Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm (Marquardt, 1963). The quasi-maximum likelihood estimator (QMLE) is used, as according to Bollerslev and Wooldridge (1992), it is generally consistent, has a limiting normal distribution and provides asymptotic standard errors that are valid under non-normality. The one step-ahead volatility forecasts of the models are

One-step-ahead forecast of the GARCH( $p, q$ ) model

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q (a_i^{(t)} \varepsilon_{t-i+1}^2) + \sum_{j=1}^p (b_j^{(t)} \sigma_{t-j+1}^2).$$

One-step-ahead forecast of the EGARCH( $p, q$ ) model

$$\hat{\sigma}_{t+1|t}^2 = \exp \left( a_0^{(t)} + \sum_{i=1}^q \left( a_i^{(t)} \left| \frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right| + \gamma_i^{(t)} \left( \frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right) \right) + \sum_{j=1}^p (b_j^{(t)} \ln(\sigma_{t-j+1}^2)) \right).$$

One-step-ahead forecast of the TARCH( $p, q$ ) model

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q (a_i^{(t)} \varepsilon_{t-i+1}^2) + \gamma^{(t)} \varepsilon_t^2 D(\varepsilon_t < 0) + \sum_{j=1}^p (b_j^{(t)} \sigma_{t-j+1}^2).$$

The ARCH processes are estimated using a rolling sample of constant size equal to 1000. Thus, the first one-step-ahead volatility prediction,  $\hat{\sigma}_{t+1|t}^2$ , is available at time  $t = 1000$ . Samples of 500 and 2000 observations were also considered leading to similar findings, thus demonstrating that the results of the simulation study are not appreciably affected by the sample size. Tables with the results for rolling samples of 500 and 2000 observations, as well as the full versions of the tables that are presented in the paper for the rolling sample of 1000 observations considered in this section can be found in Degiannakis and Xekalaki (2002).

The SPEC model selection algorithm is applied for various values of  $T$ , and, in particular, for  $T = 5(5)80$ . (Here,  $T = a(b)c$  denotes  $T = a, a + b, a + 2b, \dots, c - b, c$ ). Thus, based on the SPEC model selection algorithm, 16 agents are assumed to take part in the simulated options market. Each agent, who follows the SPEC algorithm, selects the ARCH model with the lowest sum of  $T$  squared standardized one-step-ahead prediction errors,  $\sum_{t=1}^T \hat{z}_{t|t-1}^2$ , in order to forecast next day’s variance. As in Engle et al. (1993), three more daily forecasts are added: the average of all daily forecasts, the daily minimum and daily maximum forecasts. In the sequel, the resulting forecast methods will be referred to as the AVERAGE, the MINIMUM and the MAXIMUM method, respectively.

Table 2  
The annualized daily profits per competitor per straddle for trades that are at the average of the bid/ask prices. The methods with 35 highest and 5 lowest profits are presented

Rank	Algorithm	Profit (%)	t-Ratio	Rank	Algorithm	Profit (%)	t-Ratio
1	SPEC(T = 5)	22.34	6.76	21	SPEC(T = 75)	12.02	5.11
2	SPEC(T = 10)	20.09	6.64	22	AR(4)EGARCH(1,2)	12.01	4.82
3	SPEC(T = 15)	17.94	6.53	23	AR(0)EGARCH(1,1)	11.32	5.41
4	SPEC(T = 25)	16.58	6.70	24	AR(1)TARCH(2,2)	11.04	4.95
5	SPEC(T = 20)	16.56	6.49	25	AR(0)TARCH(1,2)	10.88	4.52
6	AR(0)EGARCH(1,2)	14.44	5.49	26	AR(2)TARCH(2,2)	10.74	4.88
7	SPEC(T = 50)	14.42	6.35	27	AR(2)EGARCH(1,1)	10.69	5.31
8	SPEC(T = 40)	14.29	5.91	28	AR(2)TARCH(1,2)	10.31	4.17
9	SPEC(T = 30)	13.93	5.78	29	AR(1)EGARCH(1,1)	10.24	4.89
10	SPEC(T = 45)	13.85	5.73	30	AR(3)TARCH(2,2)	10.05	4.68
11	SPEC(T = 35)	13.80	5.69	31	AR(4)TARCH(2,2)	9.41	4.31
12	SPEC(T = 80)	13.49	5.75	32	Average	9.28	9.33
13	SPEC(T = 55)	13.10	5.56	33	AR(3)EGARCH(1,1)	9.23	4.72
14	SPEC(T = 70)	13.04	5.48	34	AR(1)TARCH(1,2)	8.94	3.53
15	SPEC(T = 60)	12.84	5.43	35	AR(3)TARCH(1,2)	8.89	3.77
16	SPEC(T = 65)	12.70	5.39	100	AR(1)TARCH(0,1)	-21.90	-6.94
17	AR(0)TARCH(2,2)	12.61	5.65	101	AR(2)TARCH(0,1)	-22.00	-6.95
18	AR(1)EGARCH(1,2)	12.54	4.88	102	AR(3)TARCH(0,1)	-22.24	-7.08
19	AR(2)EGARCH(1,2)	12.44	4.96	103	AR(4)TARCH(0,1)	-22.25	-7.02
20	AR(3)EGARCH(1,2)	12.12	4.88	104	Minimum	-37.99	-8.20

Thus, the simulated options market that has been created is comprised by 104 competitors. Each trader applies a trading strategy for the period ranging from October 4th 1995 to October 18th, 2002 on the S&P500 index, totally 1773 trading days. For 1773 trading days and 104 agents, the *i*th agent’s daily profit per straddle is computed as

$$\pi^{(i)} = \sum_{t=1}^{1081} \left( \left( \sum_{i^*=1}^{103} \pi_t^{(i,i^*)} / 103 \right) / 1773 \right).$$

Any method that yields superior profits relative to the AVERAGE method appears more suitable in predicting volatility for pricing contingent claims. Table 2 presents the profits per competitor per straddle and the corresponding *t*-ratios (ratio of average daily profit to its standard deviation divided by the square root of the trading days). The methods with the 35 highest and 5 lowest profits are presented. The agents based on the SPEC model selection algorithm clearly outperform the others. All the SPEC model selection based algorithms achieve returns higher than the AVERAGE method. The highest annualized daily returns are achieved by the SPEC(5) model selection algorithm, which is in accordance to Degiannakis and Xekalaki (2001a) results.

Moreover, the agents that employ the SPEC model selection algorithm rank at the 16 of the 22 top positions. The MINIMUM forecast takes the last positions and the MAXIMUM forecast achieves negative and statistically significant returns, an indication that neither a downward nor an upward forecast bias, that could affect profits significantly, is present. It is interesting to note that the EGARCH(1,2) and the TARCH(2,2) model selection algorithms

perform distinctly better than the remaining ARCH models. The more flexible models, which account for the leverage effect and have a higher order of  $p, q$ , outperform the parsimonious models (i.e. GARCH(0,1), TARCH(0,1) and EGARCH(0,1)). Giot and Laurent (2003), Hansen and Lunde (2001) and Vilasuso (2002), among others, have found that more flexible models beat the forecasting ability of the parsimonious ones. Of course, as the number of candidate models increases, the probability of finding models with superior predictive ability will increase as well. Note that in our simulation study, we include 3 conditional variance specifications and Degiannakis and Xekalaki (2004) have presented 31 conditional variance specifications in the context of the ARCH framework. However, the investigation of the SPEC algorithm performance with a set of more flexible ARCH models, which account for recent developments in the area of asset returns volatility, is suggested for further research.

### 5.1. Ranking of methods dropping out the least profitable agent

An interesting question to investigate is whether the performance of the SPEC algorithm is unaffected by the models that are included in the simulation. This is examined in the sequel by repeatedly running the simulation, each time having dropped out the trader using the least-profitable method and calculating the cumulative profits of the remaining participating agents. If the performance of the algorithm is not affected by the models considered, the profits of participants who trade options using the SPEC algorithm should occupy the top places of the ranking. The resulting ranks of the SPEC algorithm-based methods and the AVERAGE method are summarized in Table 3. The first column shows the number of participants in each group and the rows present the ranking of the SPEC model selection methods and the AVERAGE method within each group. As there are 104 traders, 103 groups are created, but, for space limitations, only 14 groups are presented. Although there are some slight changes in the rank, the traders based on the SPEC model selection algorithm keep the first places in the ranking. The SPEC(5) model selection algorithm achieves the highest returns in all the cases, thus indicating that the forecasting ability is not sensitive to the models that are used. On the other hand, the AVERAGE method deteriorates as the group becomes smaller. An expected feature as the sample becomes smaller by dropping out the least accurate forecasts.

### 5.2. Exercise price and relative profits

Following Engle et al.'s (1993) approach, the sensitivity of agents' profits to exercise price is examined. Table 4 shows the ranking and cumulative profits of the competitors trading 1-day straddles with exercise prices equal to  $e^{5rf_t}$  and  $e^{-3rf_t}$ . The call and put option prices are calculated as

$$C_{t+1|t} = N\left(\frac{2(1-K)rf_t + \sigma_{t+1|t}^2}{2\sigma_{t+1|t}}\right) - e^{(K-1)rf_t} N\left(\frac{2(1-K)rf_t - \sigma_{t+1|t}^2}{2\sigma_{t+1|t}}\right)$$

and

$$P_{t+1|t} = C_{t+1|t} + e^{(K-1)rf_t} - 1,$$



Table 4

The rank and annualized daily profits of the competitors trading 1-day straddles with different exercise prices

Forecasts	$e^{5rf_t}$		$e^{-3rf_t}$		Forecasts	$e^{5rf_t}$		$e^{-3rf_t}$	
	Profit (%)	Rank	Profit (%)	Rank		Profit (%)	Rank	Profit (%)	Rank
SPEC( $T = 5$ )	22.46	1	22.52	1	SPEC( $T = 75$ )	11.88	22	11.89	22
SPEC( $T = 10$ )	20.29	2	20.34	2	AR(4)EGARCH(1,2)	11.94	21	11.95	21
SPEC( $T = 15$ )	17.84	3	17.89	3	AR(0)EGARCH(1,1)	11.15	23	11.16	23
SPEC( $T = 25$ )	16.50	4	16.53	4	AR(1)TARCH(2,2)	11.03	24	11.03	24
SPEC( $T = 20$ )	16.42	5	16.46	5	AR(0)TARCH(1,2)	10.80	25	10.81	25
AR(0)EGARCH(1,2)	14.32	7	14.33	7	AR(2)TARCH(2,2)	10.79	26	10.79	26
SPEC( $T = 50$ )	14.40	6	14.41	6	AR(2)EGARCH(1,1)	10.52	27	10.52	27
SPEC( $T = 40$ )	14.21	8	14.24	8	AR(2)TARCH(1,2)	10.37	28	10.37	28
SPEC( $T = 30$ )	13.81	10	13.84	9	AR(1)EGARCH(1,1)	10.07	30	10.07	30
SPEC( $T = 45$ )	13.81	9	13.83	10	AR(3)TARCH(2,2)	10.14	29	10.14	29
SPEC( $T = 35$ )	13.79	11	13.82	11	AR(4)TARCH(2,2)	9.50	31	9.50	31
SPEC( $T = 80$ )	13.34	13	13.35	13	Average	9.46	32	9.46	32
SPEC( $T = 55$ )	13.41	12	13.43	12	AR(3)EGARCH(1,1)	9.12	33	9.12	33
SPEC( $T = 70$ )	12.89	14	12.90	14	AR(1)TARCH(1,2)	8.92	35	8.93	35
SPEC( $T = 60$ )	12.83	15	12.85	15	AR(3)TARCH(1,2)	8.99	34	8.98	34
SPEC( $T = 65$ )	12.55	17	12.56	17	AR(1)TARCH(0,1)	-22.06	100	-22.08	100
AR(0)TARCH(2,2)	12.69	16	12.70	16	AR(2)TARCH(0,1)	-22.13	101	-22.15	101
AR(1)EGARCH(1,2)	12.38	18	12.39	18	AR(3)TARCH(0,1)	-22.38	102	-22.40	102
AR(2)EGARCH(1,2)	12.32	19	12.33	19	AR(4)TARCH(0,1)	-22.39	103	-22.42	103
AR(3)EGARCH(1,2)	12.03	20	12.04	20	Minimum	-38.14	104	-38.24	104

for  $K = 5, -3$ . The rank of the traders does not change significantly. So, the cumulative profits in the simulated market are not sensitive to the exercise price that is used.

## 6. Comparing methods of model selection on simulated options

The selection of the appropriate model is one of the most challenging areas in statistical modeling. Usually, a researcher has to choose among a set of candidate models. Methods of model selection examine the ability of the models either to describe or to forecast the variable under investigation. The Akaike information criterion (Akaike, 1973) and the Schwarz Bayesian criterion (Schwarz, 1978) are model selection methods that are based on the maximized value of the log-likelihood function and evaluate the ability of the models to describe the data. In the case we are interesting in using a model for forecasting, the evaluation of the models would naturally be based on their ability to produce valuable forecasts. Loss functions, which measure either the distance between actual and predicted values or the benefit from the use of these forecasts, are used to evaluate the forecasting ability of the models. Poon and Granger (2003) reviewed a detailed record of volatility forecasting loss functions and relative references.

In the sequel, the SPEC model selection algorithm is compared with other criteria of selection that measure the ability of the models to forecast volatility again on the basis of the profits of the participants in a simulated options market. Denoting the realized at time

$t + 1$  by  $s_{t+1}^2$ , the following loss functions were considered:

1. Mean square error of variance (MSEV):

$$T^{-1} \sum_{t=1}^T (\hat{\sigma}_{t+1|t}^2 - s_{t+1}^2)^2.$$

2. Mean absolute error of variance (MAEV):

$$T^{-1} \sum_{t=1}^T |\hat{\sigma}_{t+1|t}^2 - s_{t+1}^2|.$$

3. Mean square error of deviation (MSED):

$$T^{-1} \sum_{t=1}^T (\hat{\sigma}_{t+1|t} - s_{t+1})^2.$$

4. Mean absolute error of deviation (MAED):

$$T^{-1} \sum_{t=1}^T |\hat{\sigma}_{t+1|t} - s_{t+1}|.$$

5. Heteroscedasticity adjusted mean squared error of variance (HAMSEV):

$$T^{-1} \sum_{t=1}^T (1 - s_{t+1}^2 / \hat{\sigma}_{t+1|t}^2)^2.$$

6. Heteroscedasticity adjusted mean absolute error of variance (HAMA EV):

$$T^{-1} \sum_{t=1}^T |1 - s_{t+1}^2 / \hat{\sigma}_{t+1|t}^2|.$$

7. Heteroscedasticity adjusted mean squared error of deviation (HAMSED):

$$T^{-1} \sum_{t=1}^T (1 - s_{t+1} / \hat{\sigma}_{t+1|t})^2.$$

8. Heteroscedasticity adjusted mean absolute error of deviation (HAMAED):

$$T^{-1} \sum_{t=1}^T |1 - s_{t+1} / \hat{\sigma}_{t+1|t}|.$$

9. Mean logarithmic error of variance (MLEV):

$$T^{-1} \sum_{t=1}^T \ln(s_{t+1}^2 / \hat{\sigma}_{t+1|t}^2)^2.$$

10. Gaussian maximum likelihood error of variance (GMLEV):

$$T^{-1} \sum_{t=1}^T \left( \ln(\hat{\sigma}_{t+1|t}^2) + \left( \frac{s_{t+1}^2}{\hat{\sigma}_{t+1|t}^2} \right) \right).$$

11. Gaussian maximum likelihood error of deviation (GMLED):

$$T^{-1} \sum_{t=1}^T \left( \ln(\hat{\sigma}_{t+1|t}) + \left( \frac{s_{t+1}}{\hat{\sigma}_{t+1|t}} \right) \right),$$

where  $T$  is the number of the one-step-ahead volatility forecasts. The first four loss functions have been widely used in applied studies. The heteroscedasticity adjusted functions were introduced by Andersen et al. (1999) and Bollerslev and Ghysels (1996), while mean logarithmic error function was utilized by Pagan and Schwert (1990). The GMLE function, which was presented in Bollerslev et al. (1994), measure the forecast error according to the likelihood function that is used in estimating the models.

As the actual volatility is unobservable, the common way to determine the daily realized volatility is the squared daily returns, which is an unbiased but noisy volatility estimator. Andersen and Bollerslev (1998) introduced the use of the sum squared finely sampled high frequency data as an alternative volatility measure. For a detailed description of the realized intra-day volatility, the interested reader is referred to Andersen et al. (2003) and Andersen et al. (2004). Based on Andersen et al. (1999), Andersen et al. (2001) and Kayahan et al. (2002), we compute the realized intra-day volatility of day  $t$  as

$$s_t^2 = \sum_{j=1}^{m-1} (\ln(P_{(j+1/m),t}) - \ln(P_{(j/m),t}))^2,$$

where  $P_{(m),t}$  denotes 5-min linearly interpolated prices of S&P500 at day  $t$  with  $m$  observations per day. The intra-day quotation data are available from April 28th 1997 to October 18th 2002 and were provided by Olsen and Associates.

Each loss function is computed for  $T = 10(10)80$ . In order to compare the SPEC algorithm with the 11 loss functions, a simulated options market is created. Each agent selects the ARCH model with the lowest value of the loss function in order to forecast next day's variance. The simulated market is consisting of 99 traders: the 12 model selection algorithms for 8 different sample sizes (including the SPEC algorithm), the average, the minimum and the maximum of all daily forecasts methods. The comparison is carried out on the basis of the annualized daily profits of the participants.

The resulting ranking of the criteria is summarized in Table 5 (columns 1 and 2). For each model selection criterion, the highest annualized daily profits are given (column 3) along with the values of the corresponding  $t$ -ratios (column 4) defined as in Table 2 and the sample sizes (values of  $T$ ) at which the maximum returns are attained (in parentheses in column 2). (The full table referring to the profits of the 99 traders can be found in Xekalaki and Degiannakis (2004).)

The results in the table indicate that traders who are based on the SPEC algorithm achieve the highest returns, despite the use of the realized intra-day volatility by the loss functions.

Table 5  
The annualized daily profits per competitor per straddle for trades that are at the average of the bid/ask prices

Rank	Model selection algorithm	Profit (%)	t-Ratio
1	SPEC( $T = 10$ )	16.42	3.51
2	Average	13.30	4.67
3	HAMSEV( $T = 10$ )	0.87	0.21
4	HAMAEV( $T = 10$ )	0.53	0.13
5	HAMSED( $T = 10$ )	-0.29	-0.07
6	HAMSEV( $T = 60$ )	-0.96	-0.26
7	Max	-1.18	-0.21
8	GMLEV( $T = 10$ )	-1.66	-0.40
9	GMLED( $T = 10$ )	-1.93	-0.47
10	MLEV( $T = 10$ )	-3.20	-0.77
11	MSED( $T = 10$ )	-4.01	-0.95
12	MSEV( $T = 10$ )	-4.28	-1.01
13	MAED( $T = 10$ )	-5.19	-1.23
14	MAEV( $T = 10$ )	-7.72	-1.82
15	Min	-33.42	-6.12

Moreover, the SPEC method appears more suitable in predicting volatility for pricing contingent claims, as it is the only model selection method that produces returns higher than the AVERAGE algorithm does. An interesting point is that, with the exception of HAMSEV, all the algorithms achieve their highest returns for  $T = 10$ .

## 7. Discussion

Adopting Engle et al.'s (1993) approach to comparing several variance forecast methods using an economic value criterion, the performance of the SPEC model selection algorithm was examined. Simulating an options market, in order to avoid problems related to observed actual option prices, 104 traders were assumed to trade 1-day straddles on \$1 shares of the S&P500 index, for the period from October 4th 1995 to October 18th, 2002 (1773 trading days). Traders were also assumed to use variance forecast methods of their choice. The variance forecast methods considered were: 85 selection "methods" (strategies), one for each of 85 ARCH models, each amounting to the utilization of the forecasts of the same model at any point in time, the SPEC model selection algorithm for 16 different sample sizes, the average, the minimum and the maximum of all daily forecasts methods. Traders using SPEC algorithm based methods appear to achieve higher profits than traders using any of the 85 single ARCH model based methods considered in the simulation. Moreover, traders, who apply the SPEC model selection algorithm for sample sizes  $T = 5(5)25$ , appear to achieve the highest profits, a conclusion which is in agreement to Degiannakis and Xekalaki's (2001a) findings in the case of real index-option prices. The ability of the SPEC model selection algorithm was also compared with loss functions that measure the ability of the models to forecast volatility. Even though, the other criteria (loss functions) used the realized intra-day volatility, the SPEC algorithm, for  $T = 10$ , led to the highest profits. It appears, therefore, that the results support the conclusion that the increase in profits cannot

be attributed to chance but to improved volatility prediction. Hence, the SPEC selection method offers a useful model selection tool in estimating future volatility, with applications in pricing derivatives.

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