Wind Farm with Option Interactions

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Table of contents

Introduction

Key facts about renewables investment

Overall renewables investment needs :

- Already large investments in the 2010s and ambitious renewables investment goals by 87 governments programs to add 720 GWs.
- Furthermore, corporations (e.g., RE100 initiatives) are increasingly keen to source green energy
- Unfortunately, these planned investments will fall short of reaching a reduction "“well below 2°C” as targeted under the Paris agreement.

Wind farm economics :

- Governmental incentive packages (e.g., tax credits, feed-in tariffs) to boost investment in renewables are gradually phased out.
- The economics of wind farm is becoming attractive on its own rights (e.g., lower equipment costs, increased output).
- Financial innovation has played and will play a role in promoting renewables investment !

Wind farms' financial model

- 65% of the renewables investment are financed “on balance sheets,” whereas 35% via ‘project finance’, a share likely to increase.
- In a Special Purpose Vehicle (SPV), the lending bank has no recourse against the “sponsor,” who can renege on the SPV’s obligations.
- Because of long and costly development processes (including site identification, land acquisition, permit application, project design, connection to the power grid) sometimes exceeding 10 years as well as “not-in-my-backyard” reluctance of neighboring communities, it is often easier for a sponsor to retrofit or redeploy an existing site than start a project from scratch.

Research questions

Inspired by this problem, we want to address the following research questions :

- How does the financing of wind farm on a non-recourse basis affect **operational decisions on capacity redeployment** ?
- How does the wind farm's **exposure to offtake risk** affect these decisions ?
- How do these decisions depend on **site or market-specific parameters** such as initial capacity, market growth, merchant price risk and operating costs ?
- How does **herding** and **capital constraints** affect these decisions ?

To study these questions, we model a situation where a sponsor (e.g., Equinor, Iberdrola, Ørsted or Enel Green Power) can decide to redeploy its generation capacity at a time and by a size of her choice or let the SPV die to save on fixed running costs (e.g., debt servicing, maintenance, property taxes).

Key topics in the extant, related literature

- The literature on **capital structure** discusses agency conflicts between shareholders and debtholders where the shareholders' decision is when to default (Merton, 1974 ; Leland, 1994)
- **Real options analysis** (Dixit et Pindyck, 1994 ; Trigeorgis, 1996) focuses on *timing* capital budgeting decisions :
  - Some real options paper embeds the financing of the option's exercise price via debt (Mauer et Sarkar, 2005 ; Sundaresan et Wang, 2007 ; Shibata et Nishihara, 2015).
  - It becomes increasingly common to embed other decision variables besides timing (Dangl, 1999 ; Bensoussan et Chevalier-Roignant, 2018, Trigeorgis et Tsekrekos, 2018).
  - Compound options models leverage on a set sequence of decision times. An exception is Kwon (2010) where the decision maker decides on the ordering.
- In the **operations management** literature, recent discussions about capacity choices integrate notions from real options analysis (Miller et Park, 2005 ; Sting et Huchzermeier, 2014 ; Kouvelis et Tian, 2014).

Renewables farm econo-mics

Project economics (1 of 2)

- **Output** depends on farm capacity x via a concave function $x \mapsto x^{\epsilon}$;
- **Offtake strategy** determines the extent to which revenues are stochastic. The merchant price $(Y_t^y)_t$ follows a geometric Brownian motion (see R. S. Pindyck, 1999 ; Schwartz et Smith, 2000), given by

$$\mathrm{d}Y_t = \mu Y_t \,\mathrm{d}t + \sigma Y_t \,\mathrm{d}Z_t, \quad Y_0 = y > 0 \quad \text{a.s.}$$

  under the risk-neutral measure (see Zhou et al., 2019).
- **Fixed expenses** relate to turbine and balance-of-plants O&M, land leases, property taxes, etc.

Project economics (2 of 2)

- Whether the farm can be financed via **debt** depends on the site's offtake strategy. We consider two representative cases :
  1. with power purchase agreement (PPA) and debt financing;
  2. no PPA and reduced fixed cost.
- The **sponsor's profit**, given by

$$\pi(x, \delta) := a + by^{\gamma}x^{\gamma},$$

  is not affine in the merchant price.
- The **engineering, procurement and construction** (EPC) costs is assumed fixed in a base case. Sponsors hire planning engineers and contruction companies to avoid cost overruns.
- **Stimulus programs** are ignored as they are gradually phased out.
- **Equipment's useful life** is driven by an exponential decay of the wind farm equipment. The effective discount rate $r > \mu$ accounts for such decay.

Stylized model with single options

Net present value over equipment lifetime

- The sponsor is thus entitled to the NPV

$$\psi(y,x) := \mathbb{E}\int_0^\infty e^{-rt}\pi(Y_t^y,x)\,\mathrm{d}t,$$

- If $r > \mu$, let $\beta_1 < 0$ and $\beta_2 > 1$ solve $\gamma \mapsto \mathcal{Q}(\gamma) := r - \gamma\mu - \frac{1}{2}\gamma\,(\gamma - 1)\,\sigma^2$.
- If $\gamma \in (0, \beta_2)$, the sponsor's NPV becomes

$$\psi(y,x) = A + By^\gamma x^\epsilon, \quad \text{with} \quad A := a/r \text{ and } B := b/\mathcal{Q}(\gamma) > 0.$$

- The sponsor's NPV is positive when $a \geq 0$ (Case 1).
- If $a < 0$ (Case 2), it is negative iff the merchant price y falls below the breakeven point $(-Ax^{-\epsilon}/B)^{1/\gamma}$.

Standalone walk-away option

Sponsor's problem

$$\varphi(y,x) := \sup_{\Theta} \quad \mathbb{E} \int_0^{\Theta} e^{-rt} \pi(Y_t^y, x)\, \mathrm{d}t$$

- In Case 1, $\varphi \equiv \psi$;
- In Case 2,

$$\varphi(y,x) = \begin{cases} 0, & y < y_1(x) := \left(\lambda x^{-\epsilon}\right)^{\frac{1}{\gamma}}, \\ A + B y^{\gamma} x^{\epsilon} + \frac{A\gamma}{\beta_1 - \gamma}\left(\frac{y}{y_1(x)}\right)^{\beta_1}, & y \geq y_1(x), \end{cases} \quad \text{where } \lambda := -\frac{A}{B}\frac{\beta_1}{\beta_1 - \gamma} \geq 0.$$

Further,
- the walk-away option is valuable (i.e., $\varphi \geq \psi$).
- As $\lambda \geq 1$, the sponsor waits until the merchant price is “deep in the money” to shut down.
- Greater capacity x reduces the risk of exit [i.e., $y_1'(\cdot) < 0$] because the sponsor generates more net income.

Standalone expansion option

- If $a \geq 0$ (Case 1), the capacity-choice problem, i.e.,

$$\Psi(y,x) := \sup\left\{\psi(y, x+\xi) - k\xi; \xi \geq 0\right\},$$

has an explicit solution given by

$$\Psi(y,x) = \begin{cases} A + By^{\gamma}x^{\epsilon}, & y \leq \hat{x}^{-1}(x), \\ A + By^{\gamma}\hat{x}(y)^{\epsilon} - k\big[\hat{x}(y) - x\big], & y > \hat{x}^{-1}(x), \end{cases} \quad \text{where} \quad \hat{x}(y) := \left(\frac{\epsilon B}{k} y^{\gamma}\right)^{\frac{1}{1-\epsilon}}.$$

- In Case 1, the sponsor's timing problem,

$$\sup_{\tau} \quad \mathbb{E}\Big[\int_0^{\tau} e^{-rt}\pi(Y_t^y, x) + e^{-r\tau}\Psi(Y_\tau^y, x)\Big],$$

can be solved explicitly under some parameter restrictions.

- The capacity-choice problem (and hence the optimal stopping problem) is more involved if $a < 0$

Stylized model with option interactions

Optimal capacity choice

Consider now for Case 2 where $a < 0$:

Sponsor's capacity-choice problem

$$\Phi(y, x) := \sup_{\xi \geq 0} \left\{ \varphi(y, x + \xi) - k\xi \right\},$$

- This problem is more involved because the objective function is not concave (while it is for Ψ) :

Lemma 1

The function $x \mapsto \dfrac{\partial \varphi}{\partial x}(y, x)$ *vanishes on* $(0, x_1(y))$, *increases on* $[x_1(y), x_2(y)]$ *and decreases on* $(x_2(y), \infty)$, *attaining a maximum at* $x_2(y)$.

Optimal capacity choice

The solution to the above problem will be state and parameter specific :

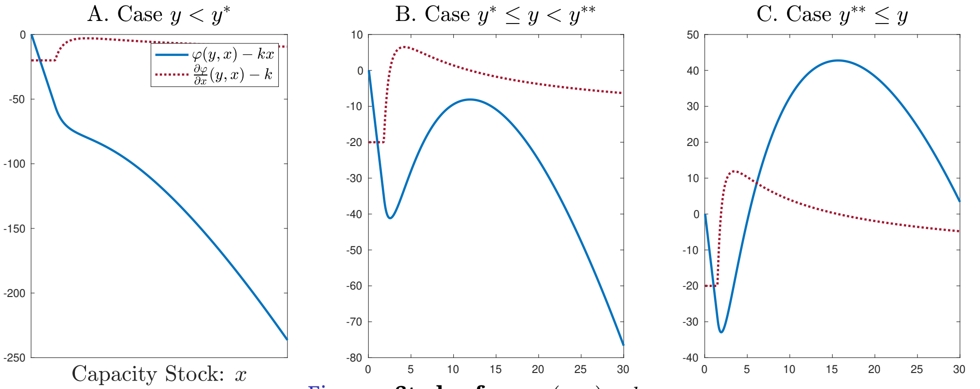

Figure – **Study of $x \mapsto \varphi(y,x) - kx$.**

Optimal capacity choice

The sponsor expands the farm's capacity from x to $x_3(y)$ iff the merchant price y exceeds a "NPV threshold :"

Theorem : Solution to optimal capacity choice problem

$$\Phi(y,x) = \begin{array}{c|c} \text{Limited existing capacity } x < x^\star & \text{Larger existing capacity : } x \geq x^\star \\ \hline \begin{cases} \varphi(y,x), & y \leq \bar{y}_3(x), \\ \varphi(y,x_3(y)) - k\left[x_3(y) - x\right], & y > \bar{y}_3(x) \end{cases} & \begin{cases} \varphi(y,x), & y \leq y_3(x), \\ \varphi(y,x_3(y)) - k\left[x_3(y) - x\right], & y > y_3(x). \end{cases} \end{array}$$

When the farm has limited capacity, deploying capacity may be used as a means to increase the disposable income to be used to pay debt, circumventing financial distress.

Important technical remark :

The gain function $\Phi(\cdot, x)$ is continuously differentiable if $x \geq x^\star$ but only continuous otherwise.

Optimal capacity choice

The positive NPV set $\mathcal{R}_a := \left\{(y,x) \in \mathbb{R}^2_+ \middle| \Phi(y,x) > \varphi(y,x)\right\}$ is depicted below :

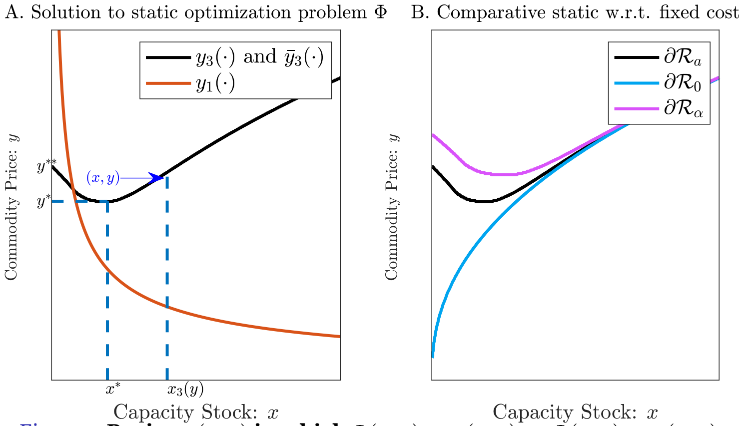

Figure – **Regions** (y,x) **in which** $\Phi(y,x) = \varphi(y,x)$ **or** $\Phi(y,x) > \varphi(y,x)$**.**

Expanding vs. shutting down

The sponsor decides when to redeploy capacity :

$$F(y,x) = \sup_{\tau} \quad \mathbb{E}\left[\int_0^{\tau} e^{-rt}\pi\left(Y_t^y, x\right)\, \mathrm{d}t + e^{-r\tau}\Phi(Y_\tau^y, x)\right].$$

The functional representation of the function F is that of the solution to the variational inequality

$$0 = \min\{F - \Phi; \mathcal{L}F - \pi\} \quad \text{for almost every } y > 0,$$

where $\mathcal{L}$ is a second-order differential operator given by

$$\mathcal{L} := r - \mu x \frac{\mathrm{d}}{\mathrm{d}x} - \frac{1}{2}\sigma^2 x^2 \frac{\mathrm{d}^2}{\mathrm{d}x}.$$

Expanding vs. shutting down

The case with large initial capacity is less daunting :

	Limited existing capacity $x < x^\star$	Large existing capacity $\delta \geq \delta^\star$
Regularity of Φ	Continuous	Continuously differentiable
Change of functions	n.a.	$\chi := F - \Phi$ and $g := \pi - \mathcal{L}\Phi$
VI	$\min\{F - \Phi; \mathcal{L}F - \pi\} = 0$	$\min\{\chi; \mathcal{L}\chi - g\} = 0$

For large initial capacity :

- The term $\chi(x, \delta)$ is the value of the "option to wait" as it is the excess value of the value function $F(x, \delta)$ above the NPV $\Phi(x, \delta)$. [1]
- $g(\cdot, \delta)$ is the economic profit from keeping the option alive, i.e., the cashflow plus the excess capital gain from deferring. We study this function analytically.

We set conditions on model primitives (which are different depending on the case considered) to ensure a connected continuation set with exit for low merchant prices and expansion for large merchant prices.

Model analysis & extensions

Change of initial generation capacity

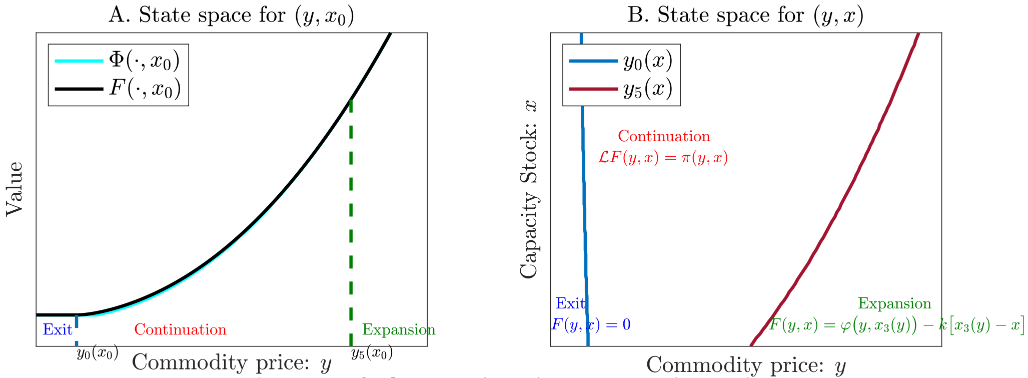

Figure – Default vs continuation vs expansion regions

Change of profitability parameters

Numerical illustration

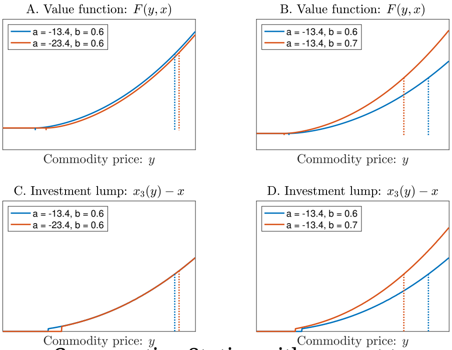

Figure – Comparative Statics with respect to a and b

Key insights

- A larger fixed cost ($a < 0$) leads to a hastened exit and a delayed expansion as well as a decrease in investment lump.
- Greater sensitivity b leads to hastened capacity expansion and a larger lump.

Change to merchant price dynamics

Numerical illustrations

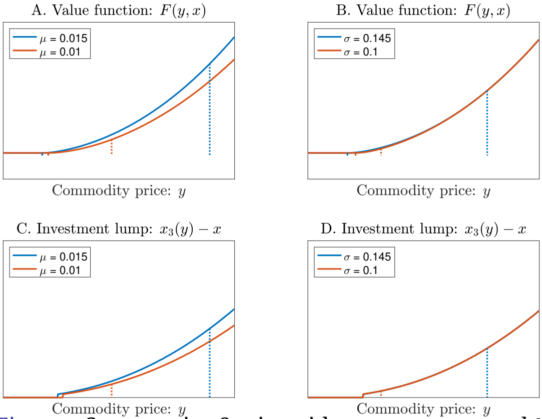

Figure – Comparative Statics with respect to σ, μ and k

Key insights

- Higher **volatility** σ enhances the option values and leads to a wider continuation set $(y_0(x), y_5(x))$.
- Higher **growth** μ delays exit because it makes the investment more attractive.

Changing EPC costs (1 of 3)

Equipment prices fluctuates over time because :

- there are continuous efforts to manufacture bigger, more powerful, and more performing equipment at lower unit cost (due to economies of scale).
- the demand for wind turbines tends to be positively correlated with power prices.

We consider a two-state Markov chain for the EPC cost $(K_t; t \geq 0)$:

- transition from $\underline{k}$ to $\overline{k} > \underline{k}$ with a probability $\lambda_{LH}(y) = \frac{2\lambda_0 - \lambda_1 y^*}{2} + \lambda_1 y \geq 0$
- transition from $\overline{k}$ to $\underline{k}$ with a probability $\lambda_{HL}(y) = \left(\frac{2\lambda_0 + \lambda_1 y^*}{2} - \lambda_1 y\right)^+ \geq 0.$

A higher intercept $\lambda_0 \geq 0$ implies more regular changes in the EPC costs, while λ_1 captures herding. We solve this problem numerically.

Changing EPC costs (2 of 3)

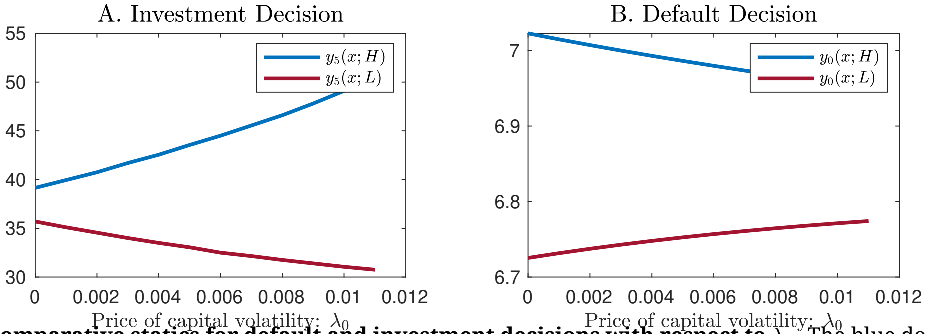

Figure – **Comparative statics for default and investment decisions with respect to λ_0.** The blue dots (red circles) corresponds to the high (low) equipment costs $\overline{k}$ ($\underline{k}$) case

Changing EPC costs (3 of 3)

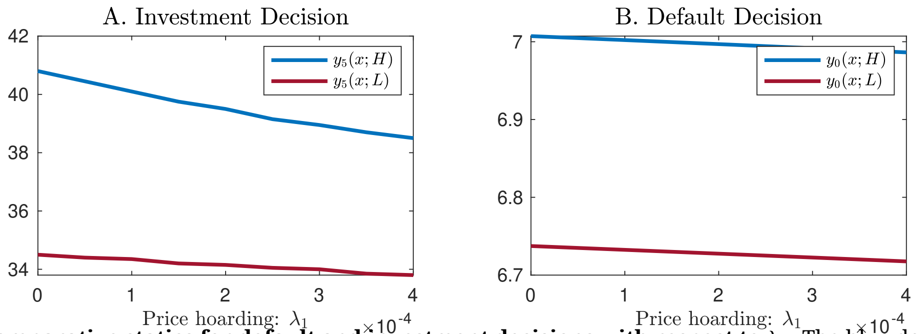

Figure – **Comparative statics for default and investment decisions with respect to λ_1.** The blue dots (red circles) corresponds to the high (low) equipment costs $\overline{k}$ ($\underline{k}$) case

Capital budgetary constraints (1 of 2)

- We implicitly assume the expansion is financed by the SPV's capital reserve.
- Consider now that the cost to raise an investment amount $k\xi$ is $\rho_0 + (1 + \rho_1)k\xi$ (see Bolton et al., 2011).
- The problem for $a < 0$ is now

$$\Phi(y, x) := \sup_{\xi \geq 0} \Big\{ \varphi(y, x + \xi) - (\rho_0 + (1 + \rho_1)k\xi) \Big\},$$

which we solve numerically.

Capital budgetary constraints (2 of 2)

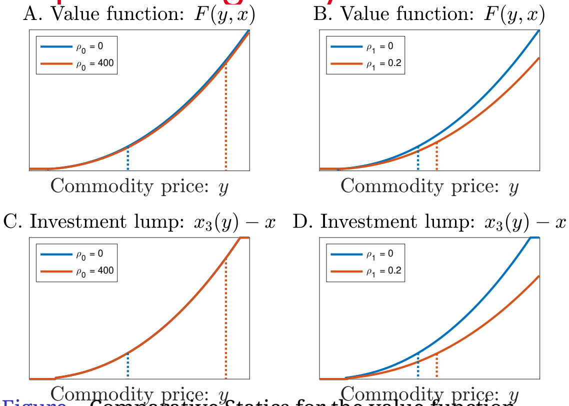

Figure – Comparative Statics for the value function $F(y, x)$ and investment lump $x_3(y) - x$ with respect to ρ_0 and ρ_1

Key insights

- If the sponsor faces a larger fee ρ_0, it delays expansion further and shuts down earlier.
- under a larger proportional cost ρ_1, the sponsor delays the expansion, reduces investment and exits earlier.

Concluding remarks

Main contributions

Managerial insights

- the sponsor installs more capacity to avoid shutting down the SPV when the merchant price is low, but disregards the exit option for larger merchant prices.
- Herding leads to an equipment price increase (resp., decrease) when the merchant price is high (resp., low), so the sponsor may hasten or delay investment to benefit from better procurement terms.
- Financing costs lead to a delayed expansion, but not to a smaller scale if costs are solely fixed.

Model limitations

- No possibility for the sponsor to upgrade the technology while renewing/installing turbines
- No consideration to reputational damage of letting a SPV go bankrupt
- No modeling of short-term (mean reverting) vs long-term price dynamics
- Output from wind farm varies (Weibull distribution).

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Appendix

Timing decisions when $x \geq x^\star$ -- Free boundary problem

We assume $\frac{\gamma}{1-\epsilon} < \beta_2$ and conjecture that the VI's solution χ solves a free-boundary problem (FBP) :

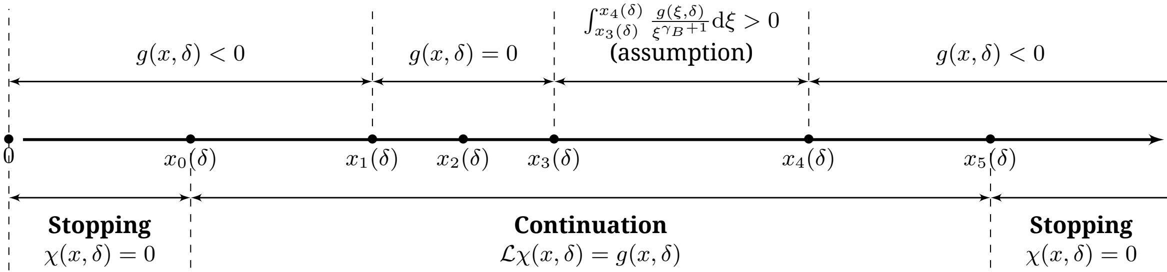

Figure – Illustration of the FBP if $x \geq x^\star$

Timing decisions when $x \geq x^\star$ -- Solution of the VI

block=fill

Theorem : Solution to the VI [if $x \geq x^\star$ and $\frac{\gamma}{1-\epsilon} < \beta_2$]

Under specific assumptions (given in the paper), the equations

$$\int_{y_0(x)}^{y_5(x)} \frac{g(\xi,x)}{\xi^{\beta_1+1}} \mathrm{d}\xi = 0 \quad \text{and} \quad \int_{y_0(x)}^{y_5(x)} \frac{g(\xi,x)}{\xi^{\beta_2+1}} \mathrm{d}\xi = 0$$

define the free boundaries $y_0(x)$ and $y_5(x)$ uniquely.
If the solution Γ to the Dirichlet problem

$$\Gamma(y_3(x),x) = 0, \quad \mathcal{L}\Gamma(y,x) = g(y,x), \quad \Gamma(y_4(x),x) = 0$$

is strictly positive, then the continuously differentiable solution to the FBP solves the VI.

Timing decisions when $x < x^\star$ -- Solution of the VI

The case $x < x^\star$ is more involved because Φ is not continuously differentiable.
We define $g(\cdot, x)$ on $(\bar{y}_3(x), \infty)$, assume again $\frac{\gamma}{1-\epsilon} < \beta_2$ and conjecture a continuation region $\big(y_0(x), y_5(x)\big)$ with

$$y_0(x) < y_1(x) < \bar{y}_3(x) < y_4(x) < y_5(x).$$

Theorem 4 : Solution to the VI

If the corresponding FBP admits a regular solution and if the solution Γ to the Dirichlet problem

$$\Gamma(y_1(x), x) = 0, \quad \mathcal{L}\Gamma(y, x) = \pi(y, x), \quad \Gamma(y_4(x), x) = \Phi(y_4(x), x),$$

then the solution to this FBP is a classical solution to the VI.